2005 Cole Prize in Number Theory

The 2005 Frank Nelson Cole Prize in Number Theory was awarded at the 111th Annual Meeting of the AMS in Atlanta in January 2005.

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years (until 2001, the prize was usually awarded every five years). The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years. These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service and as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole, contributions from Society members, and his son, Charles A. Cole. The Cole Prize carries a cash award of $5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2005 prize the members of the selection committee were: Andrew J. Granville, Richard L. Taylor (chair), and Marie France Vigneras.

Previous recipients of the Cole Prize in Number Theory are: H. S. Vandiver (1931), Claude Chevalley (1941), H. B. Mann (1946), Paul Erdős (1951), John T. Tate (1956), Kenkichi Iwasawa (1962), Bernard M. Dwork (1962), James B. Ax and Simon B. Kochen (1967), Wolfgang M. Schmidt (1972), Goro Shimura (1977), Robert P. Langlands (1982), Barry Mazur (1982), Dorian M. Goldfeld (1987), Benedict H. Gross and Don B. Zagier (1987), Karl Rubin (1992), Paul Vojta (1992), Andrew J. Wiles (1997), Henryk Iwaniec (2002), and Richard Taylor (2002).

The 2005 Cole Prize in Number Theory was awarded to Peter Sarnak. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Peter Sarnak

Citation

The Frank Nelson Cole Prize in Number Theory is awarded to Peter Sarnak of New York University and Princeton University for his work relating the distribution of zeros of L-functions in certain families to the distribution of eigenvalues in a large compact linear group of a type that depends on the family of L-functions one is considering. In particular it is awarded for the book *Random Matrices, Frobenius Eigenvalues, and Monodromy* (with N. Katz) in which this Katz-Sarnak philosophy is introduced and in which it is extensively verified in the function field case. This philosophy has had a major impact on the direction of work in analytic number theory. In addition the prize is awarded for the papers "The non-vanishing of central values of automorphic L-functions and Landau-Siegel zeros" (with H. Iwaniec) and "Low lying zeros of families of L-functions" (with H. Iwaniec and W. Luo) in which this philosophy is tested in the much harder number field case. For example, the second paper shows, subject to suitable Riemann hypotheses, that the low lying zeros of the L-functions of modular forms with root number 1 (resp.–1) are distributed like the low lying eigenvalues of a random matrix in $SO(2N)$ (resp. $SO(2N+1)$) as N gets large.

Biographical Sketch

Peter Sarnak was born on December 18, 1953, in Johannesburg, South Africa. He received his Ph.D. from Stanford University in 1980. Sarnak began his academic career at the Courant Institute of Mathematical Sciences, advancing from assistant professor (1980–83) to associate professor (1983). He moved to Stanford University as a professor of mathematics (1987–91). Sarnak has been a professor of mathematics at Princeton University since 1991 and at the Courant Institute since 2001. Since 2002, Sarnak has held the position of Eugene Higgins Professor of Mathematics at Princeton, having served as the H. Fine Professor (1995–96) and as department chair (1996–99).

Sarnak was a Sloan Fellow (1983–85) and a Presidential Young Investigator (1985–90). In 1991 he was elected to the American Academy of Arts and Sciences. With P. Deift and X. Zhou, he received the Pólya Prize of the Society for Industrial and Applied Mathematics in 1998. Sarnak was elected to membership in the National Academy of Sciences (2002), won the AMS's Levi L. Conant Prize (jointly with N. Katz) in 2003, and held the Rothschild Chair of the Isaac Newton Institute in Cambridge, UK, and the Aisenstadt Chair of the Centre de Recherches Mathématiques in Montreal in 2004. He has sat on numerous editorial boards, oversight committees, and advisory committees, and he has published extensively in the areas of number theory and automorphic forms.

Response

It is a great honor for me to receive this prize. I have mostly worked in collaboration with others. Not only has this allowed me to achieve things I could never have done by myself, but it is also more fun (especially when you are stuck, which, of course, is most of the time). This recognition belongs as much to my coworkers as to me.

In my work with Nick Katz cited above, our original aim was to determine if there was a function field analogue of the phenomenon (due to Montgomery and Odlyzko) that the local fluctuations of the distribution of the zeros of the Riemann zeta function are governed by the distributions of the eigenvalues for the Gaussian Unitary Ensemble in random matrix theory. After a lot of false starts and misunderstandings, we found such an analogue. Its source lay in the analysis of the large n limit of monodromy groups associated with families of such zeta functions. This led naturally to the possibility that the distribution of low lying zeros of a family of automorphic L-functions might also be governed in a decisive way by a symmetry type associated with the family. The extensive numerical computations of zeros of such L-functions by Mike Rubinstein, who was a graduate student at Princeton at that time, gave us valuable evidence for this belief.

The paper with Henryk Iwaniec and Wenzhi Luo, cited above, developed methods to study these questions for L-functions of automorphic forms. The paper with Iwaniec does the same for the related problem of the quantitative study of nonvanishing of such L-functions at special points on the critical line and its arithmetical applications. This allowed for the verification of aspects of the conjectured distribution of zeros as dictated by the symmetry.

One of my greatest pleasures in connection with these works has been to see how others have picked up on these ideas and run with them, far beyond what I had anticipated. Let me mention in particular the remarkable conjectures for the moments of central values of families of L-functions (Keating, Snaith, Conrey, Farmer, and Rubinstein) and the determination of some of these moments as well as far-reaching quantitative nonvanishing results for such central values (Kowalski, Michel, Soundararajan, and VanderKam).

Finally, it was Paul Cohen who many years ago, when I was a student at Stanford, pointed me to Montgomery's work on the pair-correlation of the zeros of zeta and its connection to random matrix theory and asked, why is it so?

My efforts to try to answer that question began with a paper with Zeev Rudnick on the higher correlations for zeros of the zeta function and led eventually to the works cited above.