An Interview with Peter Sarnak

Peter Clive Sarnak (born December 18, 1953) is a South African-born mathematician. He has been Eugene Higgins Professor of Mathematics at Princeton University since 2002, succeeding Andrew Wiles, and is an editor of the Annals of Mathematics. Sarnak is also on the permanent faculty at the School of Mathematics of the Institute for Advanced Study.

Sarnak received his PhD in 1980 from Stanford University under the direction of Paul Cohen, who won a Fields medal in 1966 for his proof of the independence of the continuum hypothesis and the axiom of choice from Zermelo–Fraenkel set theory.

Sarnak was awarded the Polya Prize of Society of Industrial & Applied Mathematics in 1998, the Ostrowski Prize in 2001, the Levi L Conant Prize in 2003 and the Frank Nelson Cole Prize in Number Theory in 2005. He was also elected as member of the National Academy of Sciences (USA) and Fellow of the Royal Society (UK) in 2002. He was awarded an honorary doctorate by the Hebrew University of Jerusalem in 2010.

Peter Sarnak was in India visiting Mumbai and Bangalore to deliver a series of lectures under the ICTS (International Centre for Theoretical Sciences) programme "Mathematics Panorama lectures".

B Sury and C S Aravinda from Mathematics Newsletter of Ramanujan Mathematical Society had an interview with Peter Sarnak. The contents of the interview are as follows.

B Sury: ***Before you moved from South Africa to the US, who were the teachers or peers who shaped your mathematical taste? Did your parents play any serious role in this?***

Peter Sarnak: I didn't learn about abstract mathematics until University, and in high school my main interest was in chess where I played competitively at the national and international levels. Once I was introduced to real mathematics, by a number of wonderful and inspiring lecturers at the University of the Witwatersrand, I quickly devoted all of my efforts into learning mathematics and I like to think that I continue to do so even now.

BS: Why did you think of working with Paul Cohen? Would you have worked on issues of mathematical logic if he had been working on it or, did you already know that he was working in number theory when you went to him?

PS: I had taken some basic courses in mathematical logic and even some about Cohen's technique of forcing. I had heard from some of the faculty that Cohen was a very dynamic and brilliant mathematician and found this very appealing. This information was rather accurate and I was very fortunate to learn a great amount of mathematics and especially taste and quality from Paul Cohen.

BS: ***Where do you see the future of mathematics heading in the coming years? Do you see it getting more related to physics or to computer science or to both?***

PS: I have a rather global view of mathematics; that is, there are very many interesting and active areas in mathematics. What I find very pleasing is how these often interact with spectacular consequences. There are by now many examples that one can give and some of these come not only from such interactions between subfields of mathematics but also ones in theoretical physics and theoretical computer science. In connection with the last two, in the not too distant past the flow was usually one way, namely mathematical techniques being used to prove or construct theories or structures. However, recently the impact of ideas from physics and computer science on "pure" mathematics is quite dramatic as well. I am less familiar with other sciences such as biology and their impact on pure mathematics, it will no doubt come if it hasn't already.

BS: How do you visualise yourself in the landscape of mathematics as a mathematician? Do you ever feel the need to motivate yourself?

PS: I like to work on concrete problems whose solution (or more often than not partial solution) leads to a new understanding of the mathematics that underlies the problem. Most of my work is connected with the theory of numbers where the apparent truths are often simple to state but typically very hard to prove. While some of the most compelling truths (for example the Generalised Riemann Hypothesis) have so far resisted all efforts, they serve as working hypotheses from which other striking truths follow and of which a notable number have been proven. In terms of style I would say that I like to open a door on a problem and then move on to something else. Fortunately for me, I have had many stellar students who have carried some of these things much further than I had dreamed possible.

Being a mathematician, one is almost always stuck in what one is trying to do (if not you are probably tackling problems that you know how to solve before starting…). So there are times of great frustration and at which one finds it more difficult to motivate oneself. During these periods, going back to basics and lecturing is a very good remedy.

BS: ***Do you think that professional rivalries always have a negative impact on the development of the subject or do you think they can be good sometimes?***

PS: I don't think rivalries have a negative impact. It is good that there is some competition and that people get credit and recognition for making breakthroughs. Mostly mathematicians trust each other and give proper credit where it is due, and it is unusual to see the kind of behaviour which might be considered unethical (and is more common in other sciences), but it happens.

BS: Who are the mathematicians who influenced you deeply? Are there others outside of mathematics who inspired you or continue to inspire you?

PS: Certainly for me mathematicians like Riemann and Dirichlet from the 19th century and Weyl, Siegel and Selberg from the 20th, have influenced me greatly. I still find teaching a course about some aspect of their work (even if I have done so a number of times before) to be exciting, rewarding and inspiring. So much of what we (in particular I) do relies on their deep insights and it makes one re-examine what one is trying in ones own research, giving a continued belief that there is a beautiful and complete solution to what one is looking for.

BS: Have you sacrificed some other interests while choosing mathematics as a career?

PS: It is not well appreciated outside of Mathematics that Mathematicians are working essentially all the time. It is a job in which one is doing what one enjoys (and so we should have no complaints about this!). This comes at the expense of sometimes sacrificing quality time with one's family. This problem is surely not restricted to Mathematics but all academia and is certainly worse in the industrial sector.

BS: Do you think that the mathematicians of the last century did deeper work than those in the earlier century in the fact that a number of big conjectures got solved in the last 100 years than ever before? Or is it just a culmination of those efforts?

PS: It is true that we have been lucky enough to live through a period where a number of big problems were resolved. These have involved critical ideas from different fields and so really (at least the solutions presented) could not have been done without the foundational works done by earlier workers in these varying fields. For me the mathematics developed leading up to these big breakthroughs is every bit as "deep" as the striking achievements that we have witnessed.

BS: Is it right to say that more and more number-theoretic results are proved and even discovered nowadays using group-theoretic techniques or geometric techniques (dynamics of orbits etc.)? If yes, would you advocate some changes in the order in which various subjects in mathematics are traditionally learnt?

PS: I think number theorists have no shame in that they are willing to use any techniques that will allow them to understand the beautiful truths of the field that have been uncovered, and await proof. Since many fields in mathematics were invented to attack problems in number theory, it is not surprising that these fields continue to be part and parcel of techniques that are used (for example harmonic analysis methods via exponential sums or algebra-geometric methods…). The modern theory of automorphic forms which combines various fields and captures group theoretic symmetries is especially powerful. It is not that easy to explain, or to understand why it is so. I have often wondered about someone writing a paper with the title "The unreasonable effectiveness of automorphic forms in number theory". I have my views on this and would like to hear others' views. In the context of the homogeneous spaces defining automorphic forms there is homogeneous dynamics which you mention and which has proven to be remarkably powerful in the context of questions of equidistribution in arithmetic. It is one of our growing number of fundamental tools.

BS: You are an expert in techniques from several diverse areas like ergodic theory, analytic number theory, differential geometry and representation theory. Do you think it is not sufficient to gain good expertise just in one subject to have a chance of making any significant contribution?

PS: I like tell my students that to start out you need to be a real expert in one of these fields and to make some mark on them. Then from your point of expertise, you branch out to related areas. By tackling specific problems that might require a combination of these fields you learn them quickly and before long you become comfortable with these other areas. There are mathematicians who contribute broadly and those who have done so narrowly (but in a far-reaching way). Both are perfectly good, I fall into the former category.

BS: Do you think that the Riemann hypothesis may one day be solved using some existing theories like random matrix theory when they are developed further or do you think some totally new things a la hyperbolic geometry or p-adic numbers would need to emerge for any hope?

PS: It is hard to predict how it will be solved (I believe strongly in its truth and that a proof will be found). Many of the developments (say around random matrix theory) are very suggestive as to what we are looking for. They also give striking predictions, some of which can be proven in limited ranges and are important in applications. I would like to repeat that although the Riemann Hypothesis has defied efforts of many, the subject has advanced greatly by progress which allowed us to prove a number of the consequences of the generalised Riemann Hypothesis by the approximations to it that we can prove. That is instead of climbing the big mountain we have found ways around it.

BS: Those who have performed well in mathematical olympiad competitions at the school level have often turned out to be high class mathematicians. What is the connection between possessing skills to quickly solve problems at that level and doing good research which requires sustained thinking on problems at a completely different level where quickness is not really as important?

PS: Being good at mathematical competitions is neither necessary nor sufficient to make a first rate mathematician — after all this is an ability to solve problems that are already solved — which is not the case with research. However, having such good problem-solving skills is a sign of mathematical talent and so it is surely a good thing. Moreover in recent times some of the best people (but certainly not all) doing research were stars in these mathematical competitions. I am not sure how well known it is that Perelman was wiz at these.

BS: Is there a philosophy you have of what mathematics is or what its place is in society?

PS: Not really other than it is somewhere between science and philosophy and with modern computers there is no doubt about the centrality of mathematics to much of what we do.

BS: At least in India, there seems to be a phobia among children regarding school mathematics; any suggestions to professional mathematicians to deal with this?

PS: No.

C S Aravinda: You have visited India, in particular Bangalore, twice and have also interacted with some young students from India. You may have formed some impressions of India and its mathematical tradition and culture, based on it. Could you please tell us some about your experiences, opinions and interactions during your visits? Did you find any noticeable differences between the two visits?

PS: My interactions with Indian mathematicians has always been excellent. I have learned a lot from them (including this recent trip) and I hope they have also learned something from me. A number of the best students that I have had and also interacted with are Indian or of Indian descent. At Princeton University, we have had especially brilliant students from a number of countries around the world and India is perhaps at the top of this list. So please keep them coming!

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