Science, Religion, and Tolerance

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IMPORTANT NOTE:
Instead of an index, search for any desired term using the PDF search facilities: "Edit", then "Search".

Foreword

The following pages continue my book „Science, mind and the universe: an introduction to natural philosophy" (SMU), into the topic of religion and tolerance; however, they can be read independently of that book. Primarily I wrote them to order my own ideas. Still, they might be of interest to other people who like to think along similar lines. So I decided to place the text on the Internet as a PDF file for easy download.

Since the 10 years from the time I have written SMU, I have tried to study Karl Rahner and his philosophical forerunner, Immanuel Kant, and to understand Hegel from a theological point of view. From the secondary literature I have profited most from Frederic Copleston's unconventional treatment of Kant and Hegel in his History of Philosophy (CH1, CH2) and from Michael Schulze's excellent little book on Rahner (SCH), as well as from the outstanding web page www.hegel-system by Kai Froeb, Martin Grimsmann and Lutz Hansen who represented Hegel's system most instructively as a Sierpinsky fractal. (*Abbreviations such as SMU, CO1 or SCH refer to the bibliography at the end of this paper.*)

I have been trained, by my background as a geodesist occupied with highly precise observations, to see inaccuracies and uncertainties, however small, in all observations and all definitions, however accurate. Mathematics is "absolutely" precise (as much as Gödel permits), but in its application to nature is affected by uncertainties described by the error theory basically developed by C. F. Gauss, the "princeps mathematicorum", when he performed geodetic observations.

This basic uncertainty in human thinking pervades all human language, all natural sciences in the forms of Gauss and Heisenberg uncertainties, philosophy, and even mathematics, in the form of Gödel's theorem. In my opinion, it is present also in theology: even divine revelation, when naturally expressed through the imperfect medium of human language, becomes affected by uncertainty. This may contribute to a better understanding of the problem of tolerance.

I am fully aware that my present literary attempt is also pervaded by uncertainty. It is not even professional.

I used English in which I have written all my professional books, in order to reach the international scientific community. I tried to be as simple and readable as possible, so as to address all people interested in the topic (but not necessarily practicing a certain religion). A general high-school background is amply sufficient. The present work has about the level (I hope not the character) of a good science-fiction story.

Repetitions and redundancies are intentional, for didactic reasons such as easier readability.

Detailed references are kept to a minimum because this is a personal note rather than a professional paper. Furthermore, I am an avid reader. My thinking has been shaped by my reading, but it would be impossible to give detailed references and acknowledgments. Still, I am a very grateful reader. Nowadays, the Internet has become an excellent source for serious and less serious up-to-date information.

A short list of books which I have found helpful, are recommended as additional reading at the end.

My late wife Gerlinde has been my authority on religion. I am deeply grateful to her and dedicate the present paper to her memory.

Graz, September 2005

Introduction

In January 2005 I was invited to give in Vienna a lecture on earthquakes and tsunamis. In the discussion there came up the question how a merciful God could permit such a horrible disaster as the big tsunami in December 2004 which took several hundred thousands of human lives. Being an earth scientist, I did not feel competent to venture a theological argument. I only said that, two years ago, I suffered the sudden loss of my wife. In my desperation I read the Book of Job. The result was that I felt much better. God's speech, in which He humbled Job and his friends like schoolboys, opened my eyes. Who am I to try to argue with God? In Isaiah we read: "My thoughts are not your thoughts, neither are your ways my ways—declares the Lord".

In fact, our language is not suited to express the Divine with any precision. We can talk about it only in pictures and parables.

With our ordinary language we can talk about the most common things: weather, health, food, money. Mathematics has developed wonderfully precise formulas and even introduced infinity very well. In front in God's infinity it has to be silent.

In mathematics we can prove many things. We cannot in the same way prove the existence of God. This is not God's fault, but the fault is ours: the poverty of our language and of our logic. Objects exist, but God is not an object like this tree, this house, or the few Euros I have in my pocket. It is possible to find plausible arguments for God's existence which are entirely convincing to me, but, curiously enough, not to other scientists who are much cleverer than I am.

About God's almightiness it is even possible to make cheap jokes: "if God were infinitely good, He ought to immediately abolish all illnesses, all wars and all tsunamis; He must be able to do it since He is almighty". God has not created the Evil; he has created our beautiful but infinitely complex world and our fragile freedom. If I may misuse our poor language, I should say He respects His own creation and lets it develop freely without clumsily interfering with it all the time.

And I started to think…….

Part A. Science, Logic and Mathematics

Section 1. Classical mechanics

Introduction

Classical mechanics is the logically and historically simplest prototype of a natural science. Created by Isaac Newton (1643-1727), it reigned supreme in physics for at least two centuries. Physicists tried to reduce all physics to mechanics; the materialist philosophers of the age of enlightenment tried to consider animals, and finally man, as sophisticated clockworks. This is the idea of *reductionism*: physics is reduced to mechanics, chemistry is reduced to physics, and biology is reduced to chemistry and physics. Reductionism is an excellent working hypothesis or research program: biologists are trying to investigate as much as possible by chemistry and physics; there remains enough that cannot be investigated in this way.

The popularly best known feature of classical mechanics is *determinism,* a particularly rigorous form of *causality*. If the world were completely governed by classical mechanics, the future would be fully determined by the past. This has been admirably expressed by the great mathematician Pierre Simon de Laplace (1749-1827) in his book "*Mécanique céleste"*:

> An intelligent being which, for some given moment of time, knew all the forces by which nature is driven, and the relative position of the objects by which it is composed (provided the being's intelligence were so vast as to be capable of analyzing all the data), would be able to comprise, in a single formula, the movement of the largest bodies of the universe and those of the lightest atom: nothing would be uncertain to it, and both the future and the past would be present to its eyes. The human mind offers in the perfection which it has been able to give to astronomy, a feeble inkling of such an intelligence.

This "intelligent being" has become known as *Laplace's Demon*.

If any person's future would be completely and rigorously determined, there would be no *freedom of the will.* Our own consciousness of being able to make decisions would be an illusion; we would not be responsible for our actions. Therefore, jails should be closed, judges would run out of business, criminals should not be made responsible because their behaviour is determined "by their genes", etc. This of course leads to an absurdity. The freedom of the will has, however, become a serious philosophical problem, and Kant, for whom physics was Newtonian mechanics, investigated it ingeniously.

Apart from these rather dubious implications, classical mechanics is *the* fundamental physical theory, and every course in physics will begin with it. Its importance for applications, from mechanical and civil engineering to space technology and weather forecasting, cannot be overestimated. Our life is based on it. You meet with it when you travel by railway or airplane, or when you play piano or violin.

How does classical mechanics work?

The basic notions are *force F* and *mass m.* From force we determine *acceleration a* by

$$m \times a = F \, .$$

By a mathematical operation called integration we get *velocity v,* and from *v* we obtain by another integration the *orbit* or *trajectory*: the orbit of the Earth around the Sun or the trajectory of a football.

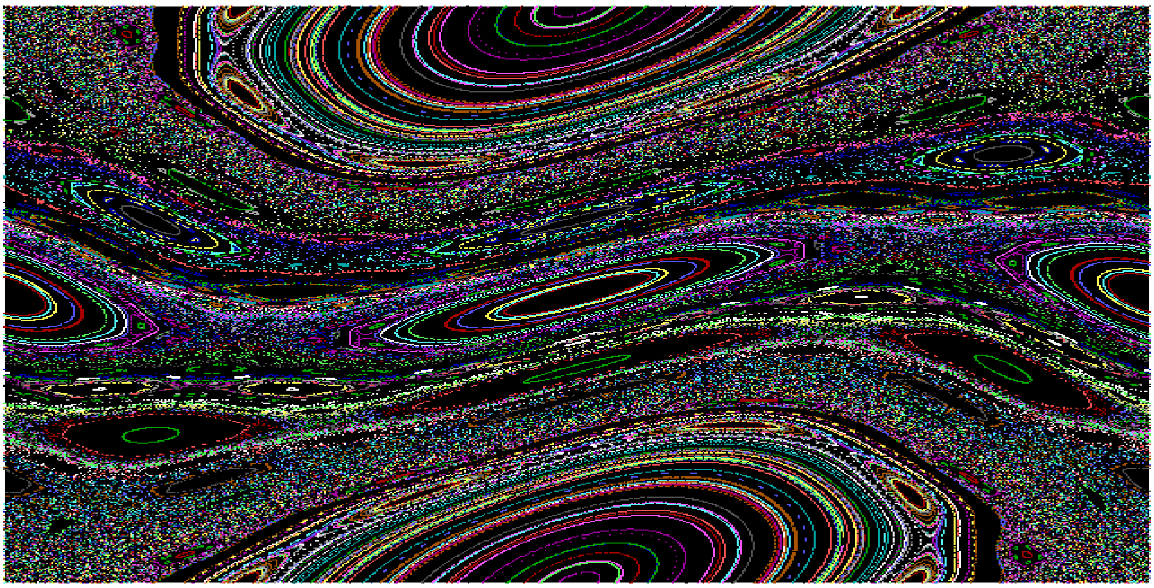
A modern application would be the computation of the orbit of a satellite around the Earth (if you are *really* interested in this topic, you may have a look at Section 7 of the book *Physical Geodesy* by B. Hofmann-Wellenhof and H. Moritz, Springer-Verlag, 2005; this is my field). Satellites have been important for telecommunication, weather forecast, and GPS navigation, to mention just a few….

A main business of classical mechanics has always been astronomy, namely *celestial mechanics.* It stood at the cradle of Newton's discovery, and it reached a culmination with the *Mécanique céleste* of Laplace quoted above.

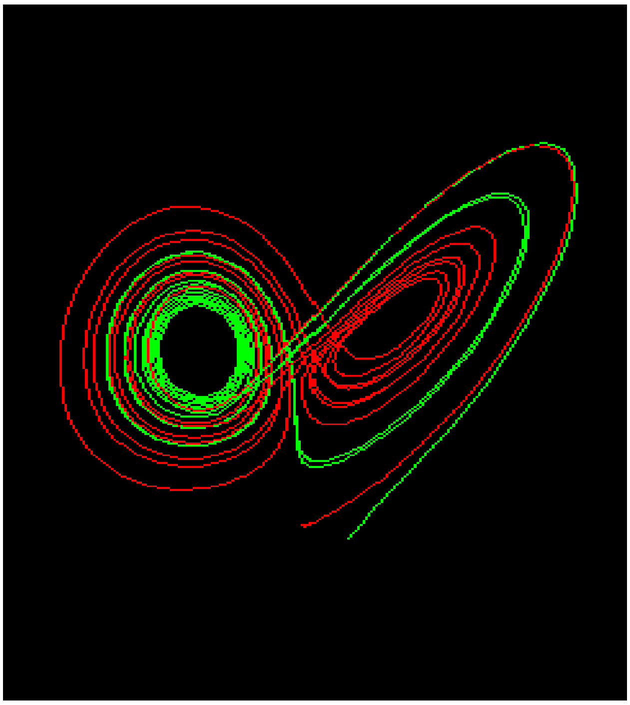
Less than 100 years later, celestial mechanics reached an unexpected anticlimax when Henri Poincarė showed that even the "eternally stable" trajectories of celestial bodies may become *instable*: the modern theory of *chaos* was born. Let us quote from his book "*Les Mėthodes nouvelles de la Mécanique céleste"* (1899):

Imagine the figure formed by these two curves and their infinitely many intersections (….); these intersections form a kind of meshwork, tissue, or infinitely dense network. (….) One is struck be the complexity of this figure which I do not even attempt to draw. Nothing is better suited to give us an idea of the complexity of the three-body problem and in general of all the problems of dynamics in which there is no uniform integral.

Only modern computer graphics has made it possible to visualize such a Poincarė meshwork:



The most famous chaos picture comes from meteorology: the “Lorenz butterfly” (1963):



What is the reason of such a complexity? It is *instability*. Laplace’s demon presupposes *stability,* which means that the future depends smoothly on the past. This is the case for the simple textbook examples in mechanics, but not generally, as Poincarė found. Instability is particularly striking in the case of the Lorenz butterfly. There are two trajectories, a red and a green one. In the case of stability, they would be close to each other all the time. In reality, however, though they are pretty close at the beginning, later on they part company completely. This is plausible for the Lorenz butterfly because of the well-known instability of weather forecasting, but it is completely unexpected in Poincarė’s three-body problem (the three celestial bodies are the sun, a planet, and its moon).

Instability implies a deviation from strict determinism, which may open even a little backdoor for *freedom of the will*. This delicate question is considered in some detail in SMU, p.245.

As we have said at the beginning, classical mechanics has been chosen as the prototype of a physical theory because of its simplicity. In the 20th century, more precise mechanical theories have been developed: the theory of relativity and quantum mechanics. Furthermore there are important other physical theories such as electromagnetism and thermodynamics.

Other natural sciences such as chemistry and biology are equally important and also have considerable philosophical and implications, for instance evolution. For this we must refer the reader to the literature, for instance to SMU, p.173.

Section 2. Mathematics

All of us have had mathematics in high school, and most of us have hated it. Therefore I will try to be as easy as possible.

Of all sciences, mathematics has always been the most exact one. All valid mathematical theorems must, and usually can, be derived from a finite set of *axioms*, at least in principle. Crudely speaking, axioms are fundamental truths which are immediately recognized as correct, even self-evident, or at least useful. For instance, they are statements such as "1+1=2" or "Through two given points there passes one and only one straight line". Euclidean geometry is based on the historically first set of axioms, which were formulated already in the 3rd century B.C.

For modern readers it may be easier to start with computers. A good mathematics program can do all you hate to do yourself: solve equations, draw ellipses, hyperbolas or any other more complicated curves, differentiate and integrate, and solve differential equations. This is done by computer *algorithms* which are built into the program. They are very powerful.

A typical formal computer language is Mathematica® by Stephen Wolfram. Mathematica® contains much of the body of mathematics that can be axiomatized or formalized. Most of mathematics can be formalized. (A complete finite axiomatization of mathematics, however, is impossible in view of the theorem of Gödel, as we shall see in Section 8. Only very few parts of physics are axiomatized, even less in chemistry and still less in the biological sciences.)

Textbooks in mathematics are partly formalized, the more so, the more they consist of axioms, theorems and formulas. Completely formalized mathematical books would, however, be unreadable. Therefore, they contain explanatory text, figures, ideas and motivations for didactic reasons. Computer languages such as Mathematica®, Maple® or Matlab® are very powerful, not only for mathematical computations but also for deriving formulas, so-called *formula manipulation*, also for the purpose of physics, space sciences (orbital computation) and in other sciences. However, as an example, even Mathematica® is accompanied by a heavy (1500 pages!) printed volume of explanations written by Stephen Wolfram in informal English, giving ample explanations, examples and instructive figures. Thus abstract *algorithmic operations,* as done by a computer, are not sufficient.

Even before computers, *symbolic or mathematical logic* was invented (*Boolean algebra*) to “formalize” logic reasoning by purely symbolic manipulations, like formulas in mathematics. A great book “Principia Mathematica” by Bertrand Russell and Alfred North Whitehead (around 1910), was written deliberately in terms of a logical symbolism as a formal deduction, but it still has informal comments and explanations interspersed: otherwise it would be unreadable.

Anyway, formulas are hardly comprehensible to most of us. Even Nobel Laureates have been implored by their publishers to avoid formulas in their popular books, in order not to lose many readers. (I myself have sinned against this with my book SMU!) Therefore, I will use formulas only very sparingly in these lecture notes; there will be only three formulas, and you may disregard them!

In the preceding section, I have mentioned Newton’s law of motion

$$m \times a = F \, ,$$

mass times acceleration equals force, and even more people will recognize

$$E = m \times c^2$$

as Einstein's famous formula: energy equals mass times the square of the velocity published exactly 100 years ago (in the "Einstein year" 1905, the "annus mirabilis").

Physics and mathematics

Both preceding formulas come from physics, and this is not an accident: theoretical physics is mathematical physics. The progress of classical mechanics has greatly influenced the progress of mathematics; its influence has culminated in general nonlinear dynamic systems (popularly called *chaos theory*), which are mathematically incredibly difficult, but numerically and graphically easily accessible by modern computers. Even young people can generate, without much explicit mathematics, the most beautiful pictures of fractals and chaotic systems. An example is the Sierpinski fractal use later in Section 14 to mirror Hegel's philosophical system. Einstein's General Theory of Relativity has greatly advanced our understanding of higher-dimensional curved spaces, and quantum theory has provided great applications for infinite-dimensional Hilbert spaces. The modern mathematical theory of elementary particles uses group theory to investigate incredibly beautiful complex symmetries.

Normal people consider these complex mathematical theories as unintelligible collections of abstruse formulas, but scientists working in these fields are overwhelmed by their almost supernatural Platonic beauty.

These theories also work perfectly in practice *but nobody knows why*. The great quantum physicist and Nobel Prize winner Eugene Wigner pointedly speaks of the "*the unreasonable effectiveness of mathematics in the natural sciences".*

The Platonic view of the world being an imperfect image of the World of Ideas, in which mathematics plays a great role, is still with us (cf. Appendix 2). Let me close with some other quotations. When Plato was asked what the main occupation of God was, he said: "God is always doing mathematics" (*Ho theos aei geometrei, SMU p.60).*

Medieval Platonic philosophers even regarded the Logos, the Second Person of the Divine Trinity, as the seat of the Platonic ideas.

Section 3. Informal reasoning and fuzziness

As we have seen, most thinking, speaking, and writing even in science is informal, whether we use our native language or the Pidgin English of international science. Formal axiomatic systems are rare.

In mathematical books, formalization is incomplete: the deductive operations between formulas are written in ordinary "informal" language; only recently, logical symbols have been introduced for simple standard logical operations. Let us also mention that, especially in books on mathematical physics and other applications, complicated logical deductions are frequently replaced by shorter and rather informal "heuristic" arguments.

The arguments we hear in daily life, in quarrels and discussions, even in university lectures, are hardly capable of being expressed in formal logic as, for instance, mathematical proofs are, at least in principle.

Curiously enough, the same holds also for the informal reasoning in philosophical arguments. Even philosophical concepts such as causality and determinism, matter and mind, freedom and law, are by no means sharply defined. They may even subtly change their meaning during a discussion.

Thus the concepts in philosophy are not usually defined in a universally accepted manner. Many of the philosophers use old concepts in a new manner, and many introduce a terminology of their own. This is particularly well seen in the case of Alfred North Whitehead (1861-1947), a philosopher thoroughly trained in mathematics and mathematical logic (see *Principia Mathematica* mentioned above!), who often refused to use standard but worn-out terminology for the sake of clarity of his own metaphysical system.

In theology there is an additional complication. Any reasoning about God introduces another almost insurmountable problem which poses a wonderful challenge. God is *infinite*, so reasoning about Him introduces all the problems and paradoxes of the Infinite as we shall see later. Furthermore, if God transcends the world, is our thinking able to reach Him at all? Here revealed religion as provided by the Bible may come to our help.

Generations of theologians have meditated for 2000 years on these problems and the result is a set of dogmas, possibly imperfect but very impressive. Can we feed all these dogmas into a computer to get all truths of faith, such as ideally we could feed all the axioms of (a limited part of) mathematics into the computer to obtain all theorems?

The obvious answer must be in the negative: theological thinking is an essentially non-algorithmic, meditative and informal reflection.

Fuzzy logic

Precise formal logic is applicable to idealized, precisely defined objects.

The real world is necessarily fuzzy. Think of yourself. Where does your body begin? How well is your skin defined? If you are a male, does a shave change the skin of your face? Does a haircut change your body? Or a manicure?

For application to “fuzzy” real-world objects, “fuzzy logic” has been invented see, for instance, SMU, p. 38. (To learn about new concepts, the fastest way may be to consult the Internet.) Gauss’ theory of observational errors is obviously the first serious mathematical theory of fuzziness; see Section 4.

Conclusion

In philosophical, theological and other discussions, statements are not simply true or false. There are arguments *pro* and *con*, some carry great weight, some arguments are rather weak. Discussions may be intended to convince the opponent, but this seldom happens. Mostly the participants in discussions and the readers of philosophical books are invited to follow the arguments, to appreciate their strength, validity, and cogency, and finally to form their own opinion.

Nobody could imagine replacing, in a discussion, the two opposing philosophers by opposing computers. So far, no philosophical book has been written (I mean authored) by a computer.

Philosophic thinking is informal for several reasons, the main being:

- It has to do with imprecisely defined concepts.
- It typically uses non-algorithmic thinking, in particular logical reflection, as we shall see in Sections 6 and 9.

This is by no means a criticism. *Creative thinking is necessarily informal.* You cannot use a computer to solve (still less to pose) philosophic problems.

Section 4. Gauss' theory of measuring errors

After earlier attempts by R. Bošković and A.-M. Legendre, it was C. F .Gauss (1777--1855) who created a theory of errors in a perfect and comprehensive form which is accepted even today, in spite of the great progress of statistics since then. The principle is that every measurement or empirical determination of a physical quantity is affected by measuring errors of random character, which are unknown but subject to statistical laws.

Error theory has always been basic in my science, geodesy. Our students learn it in the first semester, and thinking in terms of observational uncertainties, so to speak, becomes part of geodetic mentality, which has influenced also these lectures.

Let us illustrate this by two examples.

(a) Measuring the length of a house. Let the measuring result be 19.97 meters. Is this correct? We measure a second time, trying to be a little more careful; and get 19.983 meters. This is clearly different. What is the reason? There are unavoidable measuring errors and also small errors in the construction of the house itself. The architect perhaps intended the house to be 20 m long, but the construction itself was naturally not ideally accurate but “fuzzy”.

(b) We measure the three angles of a triangle. We get 50.34, 55.62 and 74.08 degrees. According to elementary geometry, their sum should be 180 degrees, but from the measurement we get 50.34 + 55.62 + 74.08 = 180.04 degrees, which is obviously a *contradiction to mathematics*. Gauss told us that *every attempt to measure a physical quantity is affected by unavoidable observational errors*. He also created an ingenious mathematical theory to take care of this case. He called it “least-squares adjustment”.

Physical uncertainties

The objects of simple and of mathematical logic are well-defined, distinct, and finite objects. They hardly ever occur in nature, because of an inherent “fuzziness” of the objects to be observed, and because of observation errors. Attempts to develop a “fuzzy logic” are on the right track but still are at the beginning, in contrast to the almost perfect Gaussian theory. The application of classical mathematical logic to nature is affected with a basic uncertainty. We will discuss this in more detail in Section 5.

Accuracy

Since observation of nature affected by errors, if the logician's alternative "right or wrong" is put to an observation such as 50.43, the rigorous answer must (almost always) be WRONG. Depending on the measurement accuracy, it might as well be 50.411 or 50.4383. Does this mean that measurements are useless? If the observation value was 250.411, it would clearly be completely wrong, an obvious blunder. The three preceding values are only inaccurate. Still, logically speaking, "inaccurate" means "wrong", and in principle, the values 50.43, 50.411, 50.4383 and 250.411 enjoy all the same logical status, which, of course, is nonsense. So, elementary logic is inadequate in such cases.

Gauss introduced the notation 50.43 ± 0.73 (say), which says that values close to each other, such as the three first) are also admissible. They are "approximately correct", the precise definition of which is given by Gaussian error theory. Measurements in natural sciences are done "within some accuracy bounds" or "error bars" but they are by no means arbitrary.

Least-squares adjustment

As mentioned above, Gauss also developed a method to eliminate the effect of the observational errors as much as possible, and to make the data mathematically self-consistent, according to a given theory or "mathematical model". This method of least-squares adjustment is based on his error theory. The details are irrelevant here. Let us, however, point out what "theory of errors" means: a "theory" of something which is "wrong", which appears to imply an internal contradiction. It does not; in fact, it is one of the great achievements of the human mind.

Least-squares adjustment, or in modern terminology, least-squares estimation, is so good that it is used even today, 200 years after its invention. Small wonder: it was invented by the "princeps mathematicorum", the prince of mathematicians.

Round-off errors and projection (only for mathematical readers)

A measurable number may well represent an irrational number. For instance, the diagonal of a square of length 1 meter has the length of the square root of 2 (in meters); this square root is an irrational number. Irrational numbers have infinitely many essentially different decimal places. Both our measurements and computations can be performed only with numbers of finitely many decimal places. So, even if there were irrational, such numbers must be rounded to a finite number of decimal places. Using mathematical terminology, irrational numbers must be projected on (a subset of) rational numbers. This terminology sounds very esoteric, but it is common nowadays and very practical. We will try it again in Appendix 2. (Irrational and rational numbers have no emotional connotation and have nothing to do with irrational or rational human behavior!)

Also all measurements and their adjusted values are, of course, rounded. Of similar character is *discretization* or *digitalization* of a continous function.

Section 5. Theory and reality: can we draw a circle?

Consider mathematical reasoning. Logical and mathematical thinking are proverbially rigorous (although theoretically limited by Gödel's theorem).

The mathematical concept of a circle is not affected by any empirical realization in nature; for instance, by drawing a circle on paper or on the blackboard. We may say, the abstract concept of a circle *transcends* any empirical realization.

To see the problem, take any mathematical theorem about a circle, e.g., its definition: the circle is the geometrical locus of all points whose distance from a given point is constant; in other terms, the circle is a curve of constant radius. (The precise value of this radius is irrelevant; in such consideration, mathematicians usually take it to be 1, which may be 1 meter, 1 cm, or 1 light year.)

Now there comes the paradox: nobody, not even the greatest mathematician, has ever seen or drawn a rigorous mathematical circle on paper or on the blackboard. Logical, mathematical, and other *axiomatic systems* are rigorous, that is, absolutely accurate, at least in principle. For instance, 2+1=3 and not 2.994. Mathematicians have discovered all properties of and theorems about a circle, without ever having been able to construct one on paper!

But what about the circles constantly used in illustrations in books on geometry etc.? They are not exact circles, as one easily sees by looking at them with a magnifying glass or under a microscope. At best, they are "fuzzy" realizations of exact, or “real", circles!

Some mathematicians write books full of geometric theorems and proofs, which do not contain a single figure. All theorems must be derivable from the axioms by logical deduction only. It is true that most such books do contain figures, but only as an aid to better visualize the geometric situation.

No mathematical theorems concerning circles can be proved completely empirically from nature. On the other hand, natural science is based on mathematical theorems. In this sense, using the terminology introduced at the beginning of this section, mathematics is *transcendent* with respect to physics. Mathematics is also *immanent* in physics, as physical formulas show; but not vice versa: mathematical theorems cannot be derived logically from physics, although mathematical theorems can be made *plausible* from physical considerations.

The fact that a mathematical circle is only an abstraction or idealization of empirical circles, occurring in nature, has been first clearly recognized by Plato in his famous *theory of ideas* (see Appendix 2).

Section 6. Logic and algorithmic thinking

The axiomatic method

The axiomatic method has been introduced already in Section 2. Let us repeat: *Axioms* are basic propositions from which all true statements of a certain branch of science or mathematics can be derived by a purely formal procedure (also by an automatic computer).

The first and best-known axiom system is Euclid's axiom system for elementary geometry (around 300 B.C). A complete and rigorous axiom system for this purpose was given, however, only by David Hilbert in 1899.

Geometric statements are fully proved also in a purely formal way from the axioms, without using intuition or figures. Figures, etc., are to be considered only as "heuristic" tools for guessing mathematical theorems or making them plausible; a rigorous deduction from the axioms must then follow. A purely formal way of deduction has been called an *algorithm*. *Algorithmic thinking* is a deduction strictly from a given set of axioms. It can in principle always be performed by a computer.

Consistency and inconsistency

It is necessary that the axioms be consistent. For instance, the possible axioms "1+1=2" and "1+1=1" are inconsistent. *From inconsistent axioms, all propositions, even logically contradictory ones, could be derived*. For instance, "2+2=4" and "2+2=3" could be derived as follows:

To ...... 1+1=2	To .... 1+1=1
add .... 1+1=2	add ..... 1+1=2
to get . 2+2=4 (true)	to get .. 2+2=3 (false)

This, of course, is nonsense because the axioms are inconsistent and the "axiom" "1+1=1" is manifestly false.

The axiomatic method accounts for the great abstractness of modern mathematics (for example, the famous French school of "Bourbaki"). Axiomatization is also a goal for other "exact" sciences as physical theories.

However, axiomatization is a final goal, but never the beginning of a science. In physics, but also in part of mathematics such as differential geometry, theorems and whole theories were at first derived intuitively or heuristically, making use of figures, additional assumptions, etc. The same holds for differential and integral calculus, which was first derived intuitively, in an inexact way (using "infinitesimally small quantities"). Only at a later stage they were made rigorous by limit processes. Many theories of physics, such as superstring theories, are still largely at a heuristic stage.

There is hardly any axiomatization for more complex disciplines such as biology, philosophy or theology.

Even the "simplest" mathematic discipline, *arithmetic* or *number theory* (the theory of the properties of natural numbers 1, 2, 3,…) *cannot be fully based on a single axiom system.* This is Gödel's theorem to be treated in Section 8.

Section 7. Logical paradoxes and antinomies

Contradictions occur even in purely mathematical or logical thinking.

Around 1900, the German logician Gottlob Frege tried to derive mathematics from logic, in order to put mathematics on a firm and exact logical basis.

Unfortunately, his (at that time only) follower, the British philosopher Bertrand Russell, discovered a paradox which made Frege so unhappy that he considered his life work useless. Russell tried to minimize the damage by finding a way to avoid his paradox, which led to the monumental work "*Principia Mathematica*", by B. Russell and A. N. Whitehead, published around 1910, as we have already mentioned.

Already the general concept of *set* (children know it from mathematics) is inherently contradictory as *Russell's paradox* shows: Let *R* be the set of all sets which do not contain themselves as members. Does *R* contain itself?

Many people, including the present author, have difficulties understanding this abstract formulation. Russell himself gave it a popular formulation which anybody can understand. In a small village there is only one barber, but a remarkable one: he shaves all male persons in the village who do not shave themselves. Does the barber shave himself? Yes, if he does not belong to the persons who do not shave themselves. The opposite is also true. Thus the barber shaves himself if and only if he does not shave himself …

A ridiculous logical children's play? Not quite, it has shattered the very foundations of logic and mathematics, a shock from which these "most exact" sciences have not recovered to the present day, and no way is seen for recovery in the foreseeable future. The very fundament of logic and mathematics, *set theory*, remains in doubt. Probably it works: nobody has found a failure yet, but this cannot be excluded with absolute certainty in the future.

A second paradox is known from classical antiquity: the *paradox of the liar.* Someone writes a sentence on the blackboard: "This statement is false". Is it correct? Yes, if the sentence is correct, then the statement holds and says it of itself. The opposite can also easily be seen. Thus, this sentence is true if and only if it is false. If we call the statement L, then L *is true if and only if it is false.*

The paradox of the liar is attributed the Cretan Epimenides, and it is even alluded to in St. Paul's letter to Titus (1: 12-13).

This paradox has been used by that logical and mathematical genius, the Austrian Kurt Gödel, to prove a highly important statement, which throws doubt not only on the absolute, all-embracing and provable exactness of mathematics, but is also basic for understanding artificial intelligence.

As we shall see in the next section, Gödel used the liar's paradox to derive his powerful theorem, replacing "wrong" by "unprovable", avoiding Epimenides' trap by a hair's breadth. Russell's paradox and Gödel's theorem have played a fundamental role in modern developments in logic and mathematics.

Antinomies (apparently contradictory statements) are used, for instance, in the dialogues by Plato, but also in the Bible, especially the New Testament, in order to express complicated matters. It is rather unfair to quote one statement out of context, omitting the balancing antithesis.

The first to make very serious systematic use of logical antinomies for philosophical purposes was Kant in his Transcendental Dialectics, which is part of his Critique of Pure Reason. His antinomies are ingenious and interesting, although they may be partly outdated by modern mathematics.

Section 8. Gödel’s theorem

Thus even logical thinking may be self-contradictory: this is most famously expressed by Gödel’s theorem. It may become relevant whenever we are trying to apply our precise thought or our precise language to *infinity*, like in mathematics. Even mathematical reasoning cannot be completely computerized, as an algorithm, as we would naively expect in view of powerful mathematical programming systems as mentioned in Section 2.

In 1931, the young mathematician and logician Kurt Gödel, then living in Vienna, published a paper with the formidable title "*On formally undecidable propositions of Principia Mathematica and related systems",*
http://home.ddc.net/ygg/etext/godel/godel3.htm .
The paper is extremely difficult, and very few people understood its importance. Nevertheless it soon became famous among specialists. *(Principia Mathematica* is the work by Russell and Whitehead mentioned in Section 7, which claimed to that all mathematics can be derived from logic.)

Gödel’s technical argument

In the preceding section we have formulated the paradox of the liar on the blackboard by writing

(L) This statement is false

and easily found:

L is true if and only if it is false.

What did Gödel do? He considered a proposition similar to (L) above:

(G) This statement is unprovable.

He then proved that G is derivable from the axioms if, and only if, its contrary, not-G, is also derivable! Thus, with "provable" being the same as "derivable from the axioms", we have

(GG) G is provable if and only if not-G is provable.

The reader will note the similarity to the paradox of the liar, discussed in the preceding section, about a proposition L: "This statement is false". We saw that L is true if and only if it is false, or in other terms,

(LL) L is true if and only if not-L is true.

Clearly, the sentence (LL) is ridiculous and pretty useless. Not so, if we consider Gödel's sentence (GG) which *differs only in replacing "true" by "provable".*

If G were provable, then not-G would also be provable. If a proposition is derivable together with its contrary, then the axioms of Principia Mathematica would be inconsistent. Hard to swallow, but possible.

There is, in fact, another possibility: neither G nor non-G is provable. Then (GG) would also be true because it does *not* say that G is provable, but only that G is provable if not-G is also provable. If neither G nor non-G is provable, fine.

At present it is generally assumed that the axioms of mathematics are consistent. Then the second alternative says that there is at least one proposition, namely G, which can never be derived from the axioms, but neither is its contrary, non-G, derivable. The proposition G is *undecidable* (see the title of Gödel's paper).

But now comes the sensation: though neither G nor non-G can be derived, it can be seen by higher-level "informal thinking" that G *must be true*. In fact, let us rephrase what we have just said:

- neither G nor non--G can be derived,
- hence, trivially, G cannot be derived, (formal deduction)
- hence, G is unprovable. (informal reasoning)

This means that the proposition G above, which says exactly this, must be true (provided, of course, that our axiom system is consistent). Clearly, this proof is not a simple derivation from the axioms but involves "meta-mathematical" reasoning. This proof is tricky indeed, but the reasoning, though oversimplified, is basically correct. From the darkness of undecidability there arises, at a higher level, the light of truth!

Thus there is at least *one* true proposition that cannot be derived from the axioms.

This is admittedly a somewhat difficult argument. (Never mind, Gödel's paper with all the details is even incomparably more difficult.)

Concluding remarks

As we have seen, deduction from the axioms is a typical activity of a computer working "algorithmically" by fixed axioms and rules of deduction. The way by which G is seen to be *true* is a typical flash of *intuition*, but no less rigorous than algorithmic deduction. However, this kind of rigorous intuition is typical for the human mind able to *reflect* "from a higher level" on the algorithmic work of the computer.

Perhaps a medical example can serve to illustrate the situation. A patient suffering from compulsory neurotic thinking, always repeats to himself a certain argument. (It is said that an ancient "philosopher" got such a compulsory neurosis by taking the antinomy of the liar too seriously, day and night repeating: L implies non-L implies L implies non-L … Had he been able to think about this “from a higher level”, he would have recognized that this argument is really nonsense, and he might have regained his normal thinking.) In fact, one way of curing a neurotic is raising his thinking to a higher level to make him recognize the futility of such an "infinite loop" of thinking.

We have used this word purposely because also in a computer there are infinite loops, which must be avoided by good programming: there are built-in mechanisms that stop the computer before an infinite loop occurs. Alas, all programmers know that computers nevertheless get sometimes into an infinite loop, and often it may be necessary to shut down the computer and start it again… Whereas the loop (LL) is deadly but irrelevant, Gödel's formula (GG) is logically acceptable and incredibly fruitful.

Two results of Gödel's theorem should be pointed out.

- Even mathematics cannot be completely derived "algorithmically", although computer algorithms are very useful, not only in numerical computation but also in computer algebra and computer logic (e.g., theorem proving). Hopefully, mathematics is consistent; to the present day no case to the contrary seems to have been found. However, we can never be absolutely certain; an element of "Gödelian uncertainty" remains.
- Computers working algorithmically can never be intelligent in the way humans are, because they cannot reflect about themselves, about their own thinking: they cannot display "creativity" or "intuition". To repeat, "intuition" in the sense used by Gödel is to recognize as true a proposition that cannot be derived from the axioms. There is nothing mystical in this, and it is as rigorous as algorithmic thinking.

Thus, computers can work only "algorithmically". Man, in addition, can think "non-algorithmically". ("Non-algorithmic thinking" is but another expression for "intuition" or "creativity", but it sounds less mystical.) Since computers cannot think non-algorithmically, they can never replace human thinking. “Artificial intelligence” can never replace “human intelligence”. This point has been emphasized recently in great detail by the mathematician Roger Penrose.

Section 9. Multi-level and dialectic thinking

Gödel's proof shows something which is absolutely remarkable: in contrast to a machine, man can think simultaneously "at two levels": a lower level, "algorithmic thinking", is accessible to computers as well as to humans, but a higher level, "non-algorithmic thinking", is reserved to man only.

Other instances of "non-algorithmic thinking" are "reflexive thinking","self-referential thinking", as well as "self-consciousness" or "creativity", even "meta-thinking".

This thinking in two levels is not a useless hair-splitting, but the basis of Gödel's proof, which has been seen to have enormous theoretical and practical importance for artificial intelligence. By replacing "true" by "provable", Gödel has tamed the destructive energy of the paradox of the liar, turning it into a highly sophisticated logical proof (some people regard Gödel's proof as the most important single achievement in mathematical logic and Gödel himself as the greatest logical genius of all time, with the possible exception of Aristotle).

Such a thinking "at two levels" occurs whenever I reflect about the possible value or insignificance of my latest scientific work (“This was a stupid mistake“; “This looks alright to me”). Such self-critical thinking is impossible to a computer. A computer will never spontaneously write on the screen "Thank you, dear programmer, your program has been really great" or "It is a shame that I must work with such a stupid program".

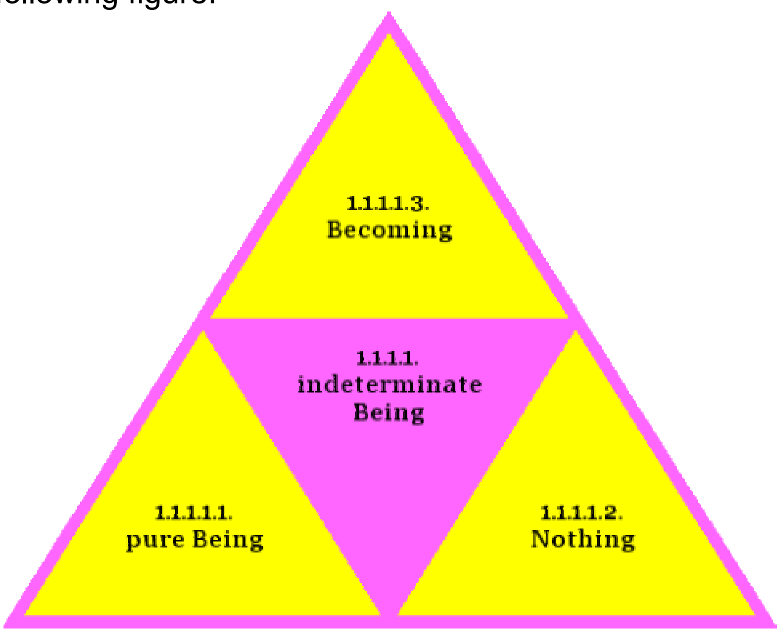
Multi-level thinking is quite common in philosophy. One of the most famous philosophical statements is "Cogito, ergo sum", "I think, therefore I am". This conclusion is *not* a deduction of formal logic, which could be done algorithmically by a computer. Instead, the conclusion follows by reflecting on the meaning of the fact that I am thinking, by reflecting on thinking at a higher level. This cannot be done by a computer! No computer algorithm would accept “cogito” (I think) as input and give “sum” (I am) as output. By the way, this is perhaps the simplest example of "non-algorithmic thinking" and thus may help understand Gödel's argument. In fact, we may say: with Descartes, from low-level thinking or even doubting ("cogito" or "dubito"), there follows higher-level certainty of existence ("sum"). With Gödel, from low-level undecidability there followed higher-level truth. To repeat, Descartes' thinking is rather a reflection on the meaning of "cogito" to obtain "sum", it is "non-algorithmic thinking".

Another example, though less known, has also played a great role in philosophy. It is due to the Greek philosopher Plotinus (around 200). He formulated the statement "The thinking thinks the thinking". You may say: "Of course, what else?". But try to program this statement in a computer! As far as I know, this statement cannot be formulated in any known computer language but if it could, a horribly destructive infinite loop might follow. We know the reason: a computer can work at one level only, whereas Plotinus' sentence comprises no less than three logical levels: one for the subject "The thinking", a lower level for the verb "thinks" and a still lower level for the object "the thinking".

Philosophic thinking is typical logical reflection or "dialectic thinking". This was clearly recognized already by Plato (dialogues Parmenides and Timaios, see Appendix A1.3) and used with perfect virtuosity and brilliance by Hegel. Hegel's famous "Logic" is dialectic logic. For an introduction to dialectic logic in connection with mathematical logic see SMU p. 44.

Characteristic for dialectic thinking is reflection, thinking on my own thinking. For instance, I may recognize that I have said nonsense. This, and Decartes "cogito, ergo sum", are typical cases of dialectic reasoning.

Hegel is thinking in consecutive triads. However, the standard formula, *Thesis, Antithesis and Synthesis,* has been popularized by the Marxists; Hegel seldom uses it (CO2, p.177). For instance, *being* ("pure being" is the thesis, non-being or *nothing* is the antithesis, and *becoming* is the synthesis: the transition from non-being to being; see the following figure:



For other simple and colourful examples see the web page www.hegel.net/en , from which we have gratefully borrowed this picture with permission. Again, the Internet is highly recommended. See also the part of Section 14 devoted to Hegel. A modern reconstruction of Hegel's system has been given by the mathematician Andreas Speiser ("*Elemente der Philosophie und der Mathematik*", Birkhäuser, Basel, 1952).

Part B. Philosophical

Section 10. The basic subject-object structure of experience

The Critique of Pure Reason by Immanuel Kant shows that the subject is involved in all knowledge. This is Kant's famous *subject-object structure of knowledge.*

This is easily seen. I cannot observe the Universe without including myself, the observer. A trivial example is the observation of a dog, whose behaviour is certainly affected by my observation: he tries to jump at me to play with me or to bite me. A more sophisticated example from modern physics is Heisenberg's uncertainty principle: observing an electron by light, that is, by a photon, leads to an unpredictable disturbance of both particles on collision.

From this basic subject-object structure of experience, Kant concludes that the universe in its totality cannot be considered as an object without observer, as is would be natural to think for a scientist.

Most people would like to be objective. Scientists like to deal with the *objective world*. What is this? Well, the world as it really is, a world with all traces of subjectivity removed, a world without observers, so to speak.

This would be a world without *human* observers. Why not? In the times of the dinosaurs, no human being existed, and it would be ridiculous to assert that in those times, the world did not exist.

The doctrine that only the real, objective world exists and is independent of any observing subject, is the great doctrine of *materialism.* Curiously enough, it immediately leads to as contradiction, as pointed out emphatically, perhaps for the first time, by Kant.

Well, I am modest enough to recognize that the world would go on if I would not exist, as it will go on after I have died. If I am the Subject of philosophy, my poor "I" really does not matter. If I am cut out of the picture, other people are ready to take on the role of the observing subject. However if, in a thought experiment, you successively eliminate all observers, one after the other, you will be finally left with a world without human beings! (This picture is not completely unrealistic in view of an all-out worldwide nuclear war, but we are not counting on such an unfortunate event.)

So, under the present circumstances of existence of human observers, such subjects are necessary, and materialism is not a reasonable alternative. (After the end of humanity, the situation will be different again.)

Thus, the subject-object structure of experience plays an essential role. According to Kant, *the observable universe cannot be an object.* (To repeat, the word "observable" is essential.)

Such reasoning by contradictions, or *antinomies* as Kant called them, is typical for this great philosopher. He introduced the terminology, *transcendental* reasoning, for this type of arguments (see also end of Section 11).

Section 11. Kant's conditions *a priori.*

Kant further analyzes the basic subject-object structure of experience. What is the effect of the subject? It poses some *conditions a priori affecting any possible observations.*

To give a first simple example: Imagine that all people are wearing red-tinted spectacles. They would conclude that all things were red.

Another famous example has been given by Eddington. A marine biologist is exploring the life of the ocean. In order to get specimens of animals living in the sea, he throws a net and examines the catch. He discovers:

- No sea animal is less than 5 cm long.
- All sea animals have gills.

In order to make sure that his discovery is correct, he repeats this experiment many times at various places. His laws are always confirmed, so that he concludes that they are universally true.

It is obvious that at least the first "law" is not an "objective" law of nature, but a "subjective" consequence of the experimental setup. It would have been different if he had used a net of smaller, or larger, mesh size.

Another important case is the three-dimensionality of space. According to Kant, we can only perceive three dimensions, so even if "real" space were higher-dimensional, we could imagine only three dimensions ("project the world onto three dimensions"). It seems that the three-dimensional character of space is "*given a priori*" by the subjective structure of our thinking. (In fact, quantum physicists work with infinitely-dimensional "Hilbert space", which is an auxiliary tool and not a "real space".)

Every perception not given a priori is called "a posteriori". The main example is *sense perception* or other empirical observations.

To give a simplified example: the great theories (classical mechanics, relativity and quantum theory) and their basic formulas which you can find in textbooks may be considered *a priori*; their observational background originally was *a posteriori*. These theories form an "a-priori" reference for further "a-posteriori" empirical investigations.

Kant was right about the three-dimensionality of space but wrong in that space is Euclidean because, according to Einstein's General Theory of Relativity, space (or rather space-time) is curved.

Still, Kant's theory about structures a priori and a posteriori and, slightly differenty, *analytic* and *synthetic* structures has proved highly fertile and influential on philosophical thinking of such different thinkers as Hegel and Bertrand Russell.

Analytic propositions can be logically deduced purely logically from a set of axioms; *synthetic propositions* cannot be obtained in this way. Roughly, *a-priori propositions* are analytic, and *a-posteriori propositions*, obtained "empirically" from observations, are synthetic. But this is not quite true.

Kant's famous question is: "Are synthetic a-priori propositions possible?" Kant thought that *mathematics* is synthetic a-priori: it is given a priori, without presupposing sense perceptions, but is not analytic because it cannot be derived from a (finite) set of axioms, as Gödel shows.

Russell thought that mathematics can be completely derived from logic, that is, that is analytic. This is what his book “Principia mathematica” tried to show around 1910. Two decades later Gödel proved his negative theorem (Section 8) on the very basis of “Principia mathematica”! So Kant’s point of view concerning mathematics seems to prevail after all; see Appendix 2.

According to Kant, there is no “pure observation” of “objective reality” without influence of the subject. “Objective reality” or Kant’s “Ding an sich” (thing-in-itself) is unknown in principle. The subject or the observer or the observational system is always involved. Every observation depends on the object *and* on the a-priori structure of the subject.

The dependence of the observation is called by Kant the *empirical* element of the observation. This corresponds to usual language.

This dependence on the subject is called by Kant the *transcendental* element of each observation. On a first glance, this is a very confusing terminology. It needs some explanation.

Transcendental vs. transcendent. This is an important distinction. “Transcendent” means “outside the universe” or “above the universe”. It is the contrary of “immanent”, which means “inside the world”. In this sense, God may be transcendent or immanent, or both.

The term “transcendental”, as introduced by Kant, has a completely different meaning. It is the contrary of “empirical”. Kant says: “I call all knowledge *transcendental* which is occupied not so much with objects as with our mode of cognition of objects, so far as this is possible *a priori* ” (quoted after CO1, p.231). Thus one speaks of transcendental knowledge, transcendental philosophy etc.

The term “transcendental” has been introduced into theology by Karl Rahner.

Section 12. Infinity in mathematics, philosophy and theology

Mathematics

Again we take our familiar example, the *circle* (of radius 1 meter, say). Is it finite or infinite? It is both, but in a different way. If we consider its *extension*, it is *finite*; for it comfortably fits a finite square of 3 meters by 3 meters, say. Considered as a *set of points* of constant radius, however, it is *infinite:* this set consists of infinitely many points.

In mathematics, Georg Cantor has developed a theory of infinite numbers. Basically, there is the *countable infinity,* formed by the set of all integers (1, 2, 3, 4,…..) and an *uncountable infinity*, which is essentially greater, consisting of the set of all the points of the circle.

Physics and philosophy

Kant's first antinomy proves as a thesis that the world has a beginning in time, and as its antithesis that it has been lasting forever. Mathematics has given a remarkable solution of this antithesis. Instead of time t we introduce a new time variable T by

$$T = \ln t,$$

where ln denotes the *natural logarithm*, Then $t = 0$, the begin of the universe, the "big bang", corresponds to T = - Infinity, which corresponds to "no beginning" in terms of T.

This shows that mathematics can help philosophy solve some of its basic problems. Unfortunately mathematics cannot solve our problem of the existence of God, which is much more complex!

(By the way, this is our third and last formula in this review, and you need not understand it. Just admire it!)

Even the question whether the extension of the universe is finite or infinite, admits of a similar mathematical treatment, the details of which are given in SMU, pp. 126-127.

It is similar with the classical philosophical paradox of *Achilles and the tortoise*: though Achilles runs much faster than the tortoise, he can never overtake it. Mathematically, the solution reduces to the summation of a simple geometric series (*ibidem*, p. 128), which results in a finite number.

Theology

There is a general agreement that *God is infinite.* The preceding considerations have shown, however, that there are many kinds of infinity, and that one must be very careful to distinguish them. Hegel has attempted it: he regarded the mainly transcendent God of the Old Testament as a wrong idea, a "bad infinity", and he considered the trinitary God of Christianity as a right idea, a "good infinity" (CO2, pp. 165-166). This is not very clear, however.

It would be very desirable to have a much more precise concept of "God's infinity". It is highly doubtful whether this is possible. Thus we are lead back to the Introduction. Infinity is indeed a very complex and elusive concept.

A simple result can be obtained easily. The Christian concept of *trinity*, one God in three divine persons, has been attacked for *tritheism* (three gods), incompatible with monotheism (one god). The argument is that, mathematically, 1 is not equal to 3. This argument, however, holds only for a finite God. For an infinite God we may apply Cantor's theory of transfinite numbers,

1 × infinity = 3 × infinity,

which is perfectly correct, even for the simplest mathematical concept of infinity. Of course, this in not a "mathematical poof of the trinity", but only an indication that reasoning about infinite beings should not be too simple-minded.

Part C. Some remarks on religion

Section 13. God and existence

Whether we believe or do not believe in God, we would be in considerable logical difficulty to define God. God is not an object like a house or a tree for which our language was developed originally. Still, speaking of God, even to deny His existence, we associate a pretty universal intuition about God: He is almighty and infinitely good, and, usually, He is considered the creator of the universe—if He exists. Trees exist, you and I exist (hopefully), and in another, rather precisely defined sense, mathematical objects exist (or not). God, however, is neither a material object, nor a human person like you and I, nor even a mathematical object, whatever Plato may have thought.

If we try to extend the concept of existence to God, we meet with the formidable difficulty that He is "infinite in all directions" and therefore beyond the reach of our language. So the fault is not with God but with the inadequacy of our language!

In fact, neither "God" nor His way of "existence" can be scientifically defined, and still most of us think that the question "Does God exist?" is not meaningless. Religious persons may answer "Yes", atheists believe they have a right to say "No", and frequently the answer is "I don't know".

Many philosophers from Aristotle onward have felt it is the job of philosophy to "prove the existence of God", whatever it means. Another problem is to show that the God of the Bible is the same as the "God of the philosophers".

We all agree that God, if He exists in some way, must be infinite. No wonder that His existence cannot be proved "exactly", that is, by formal logic. There are also well-known cheap paradoxes of the kind: "If God is almighty, He must be able to create a stone so heavy that He cannot lift it up. If He cannot lift it up, however, then He is not almighty." Here infinity is hidden in the ambiguous term "almighty".

On existence proofs

Let us try to see whether these considerations have an effect on our understanding what a proof of God's existence means, at the risk of some repetition.

In view of the importance of the problem, let us first summarize what we know on formal and informal reasoning, reformulating it in other terms.

In the precise language of modern logic, a scientific proof is a logical deduction from some well-defined set of axioms, such as a mathematical proof. In principle, it could be done by a computer algorithm. If there is such a proof, we say that the mathematical object under consideration has been shown to exist. For instance, the solution of a mathematical equation exists in this sense (or not).

Even in a science it is rare that it is completely axiomatized by a finite set of generally accepted simple axioms. The axiomatization of a science is the final stage rather than the beginning of a science. Euclidean geometry has been fully axiomatized only by David Hilbert a hundred years ago, and it is a relatively simple “science”. In 1931, Kurt Gödel showed that even arithmetic, the mathematical theory of integers or whole numbers 1, 2, 3,…, cannot be axiomatized by a final set of axioms. In fact, as we have seen, even the consistency of arithmetic cannot be proved! (There is a joke, and a pretty intelligent one, that God exists because mathematics is consistent, and the devil exists because we cannot prove its consistency.)

The root of all problems in formal logic is *infinity*; already the simple fact that the set of all integers 1, 2, 3,… is infinite, creates problems. Try to feed some infinite input into a computer; it will not work, or it may put the computer into an infinite loop. With infinite sets, we can generate all types of logical paradoxes or antinomies. Even Gödel’s proof has been seen in Section 8 to be based on the ingenious use of a paradox similar to the well-known paradox of the liar: I say: “What I am now saying is false”.

What they show is that our language, whether formalized by mathematical logic or just plain everyday speaking, is restricted to very simple objects such as trees, houses or, with some reservation, to the most simple objects of mathematics like circles or straight lines or sets of a *finite* number of integers, say. The general concept of set introduced in high-school mathematics is already beyond simple languages as Russell’s paradox shows (Section 7).

Exact (formal) logic is thus restricted to simple finite natural objects: to houses or trees, or simple geometric objects such as points, circles, or triangles. But even here we have basic problems: our material objects are not precisely defined, they are “fuzzy” and change with time: consider a cloud. Still we would like to be able to speak of clouds. Cloud arithmetic is funny: 1 + 1 = 1: two different clouds may coalesce with each other to form one cloud.

Simple mathematical objects do not occur in nature: there are no points, only ill-defined fuzzy “dots”. So we always must “idealize”, as the books on physics say (or take for granted). There are no “infinite” objects; we always use “finite approximations”. Since Gödel we know that even mathematics is not always “completely precise”. So there are uncertainties even in the most precise science, mathematics.

Ordinary language is extremely imprecise, as any conversation or discussion shows. It is indeed a poor tool for philosophical reasoning: Trouble arises if nevertheless language is treated in an over-precise way, which easily leads to contradictions or even to permanent misunderstandings.

The same holds for the use of mathematics in empirical sciences, where the sum of three measured angles in a triangle is almost never precisely 180 degrees as it should. Therefore, before applying mathematics to measurements, we must submit them to some idealizing treatment, least-squares adjustment, invented by Carl Friedrich Gauss, the “Princeps Mathematicorum”, as we have seen in Section 4.

Furthermore, what does “existence of God” mean? He is not an object of the same type as a house or a tree or another human person. Nor is He a mathematical object. If we restrict the term “existence” to these types of objects, God certainly does not “exist”. Still, the “existence of God” in some way has an intuitive meaning and is meaningful to many people, including myself.

So the reason for the fact that God cannot be “proved exactly” is the inadequacy of our language. It was the merit of Kant to have investigated this with incomparable thoroughness. He has freed the idea of God from too direct “metaphysical compliments paid to Him”, as Alfred North Whitehead said.

Section 14. Kant and German Idealism

From ancient times up to Descartes and Leibniz it was considered a main business of philosophy to prove the existence of God. It is a commonplace that Kant destroyed the metaphysical illusions about such “proofs”. Let us try a brief and very simplified review.

Kant

The best discussion on the topic of such existence proofs in a truly philosophical spirit, carefully and objectively on the very highest level, is still that of Immanuel Kant. After 200 years, it is as fresh and impressive as ever. True, Kant is not easy to read, but a very objective and readable introduction can be found, for instance, in Fr. Copleston’s History of Philosophy, vol. VI (London 1960, many reprints), denoted in our usual way by CO1 in the references at the end of the paper.

A first brief summary might be as follows.

- In his “Kritik der reinen Vernunft“ (Critique of Pure Reason), Kant shows that the existence (or non-existence) of God cannot be directly established by means of “pure” logic or empirical science. Crudely speaking, God is not an object of natural science. More generally, metaphysics is not a natural science.

- In his “Kritik der praktischen Vernunft” (Critique of Practical Reason) he thinks it is pretty convincing to approach the problem from the “practical” side of moral laws.

- In his “Kritik der Urteilskraft” (Critique of Judgment), Kant believes that the “design” of the universe, as represented by the apparent purposefulness of biology (in modern language, of evolution) may be capable of giving an indication of God as “Designer of the universe”.

It is outrageous to try to summarize the extremely careful and painstakingly cautious arguments of Kant by such a reckless oversimplification, but it may give a first vague idea of the problem. The reader should at least try to read CO1 or some other good introduction to Kant’s philosophy.

Kant: The moral law

Kant thinks that the argument form the moral law within us is the strongest. Kant implicitly considered the natural moral law (as expressed, e.g., by the Decalogue) as evident to every person of good will.

Kant's definition of the moral Good by his well-known *categorical imperative* has been criticized as being too rigid and formal, omitting human sympathy and love. Here the New Testament (especially the Sermon on the Mount) would present an ideal complement. In this way, ethics and religion could really support each other.

Kant: The beautiful design of nature

Explicitly or implicitly, a scientist, trying to discover natural laws, must start form the working hypothesis that nature is an intelligible whole covered by a universal order. An element of randomness or chaos is not excluded, but it does not look as if it were *all* random, without meaning and purpose. It is difficult or even impossible to explain what the purpose is, but it seems to be there. In a similar way, the beauty of a rose or of a work to art cannot be completely explained, but it seems to be there, if not "objectively", then "inter-subjectively", for the experts. It is very difficult for a creative scientist to remain indifferent with respect to the beauty of laws of nature. The structure of modern physics, especially relativity and quantum theory, is extremely beautiful to people who understand it. In fact I feel that the mathematical structure of Einstein's theory of relativity is as beautiful as St. Peter's Cathedral in Rome or as a symphony by Anton Bruckner.

The beauty of mathematical laws of physics *comes from the perfection of mathematics.* This is an old idea which goes back to Plato. See Appendix 2, particularly what will be said on World 3.

It is stroke of genius that, in his Third Critique, Kant treated aesthetic beauty, as represented by a flower or by a work of art, on the same footing as purpose and "design" of the universe as represented by an animal or by evolution. So we naturally come to the concept of a universal Designer. This is the famous "argument from design".

Note that such an aesthetic argument, based on the perfection of scientific laws, does not have itself the logical status of a scientific law. It is rather obtained by *"transcendental" reflection on this perfection,* in the sense of Gödel's law or Descartes' "Cogito, ergo sum" of Section 9. So the Third Critique does not contradict Kant's negative result of his First Critique.

Pointedly we may say that *God's existence is not a law of nature but may be inferred by a transcendental reflection on the laws of nature.*

Anyway, assuming a high degree of intelligible order is a necessary condition for a scientist's work, which would otherwise be meaningless. Great scientists particularly appreciate this. In the Einstein year 2005 it is appropriate to quote Einstein: "I want to know how God created the world". "God" here need not be a personal creator of the world, but a kind of "cosmic world order". This seems also the thinking of Plato; A. N. Whitehead said so explicitly.This is, so to speak, a "minimum concept" of God. As a matter of fact, He is by no means excluded to be the Creator, but this question seems to be beyond philosophy. Generally it is the old theological problem whether the “Biblical God” can be at all identified with a “God of the philosophers”; this was the question of Blaise Pascal. The physical “Big Bang” might even be more helpful for a theological “theory of creation” than philosophy. Curiously enough, general evolution might be more relevant, even if one is not prepared to go to the extremes of Pierre Teilhard de Chardin. I have said this here to show how much Kant had anticipated the modern situation: he had foreseen that his Third Critique would become most relevant for theology.

The First Critique again

One of Kant’s most important contributions is the analysis of the basic subject-object structure of experienced reviewed in Sections 11 and 12. Already Kant has introduced the “transcendental subject” as the abstract idea underlying all empirical subjects. Its implications on theology have been recognized by Fichte and thoroughly investigated by Hegel. Recently, it was considered from a completely different perspective, by the great theologian Karl Rahner; see Section 15.

Fichte

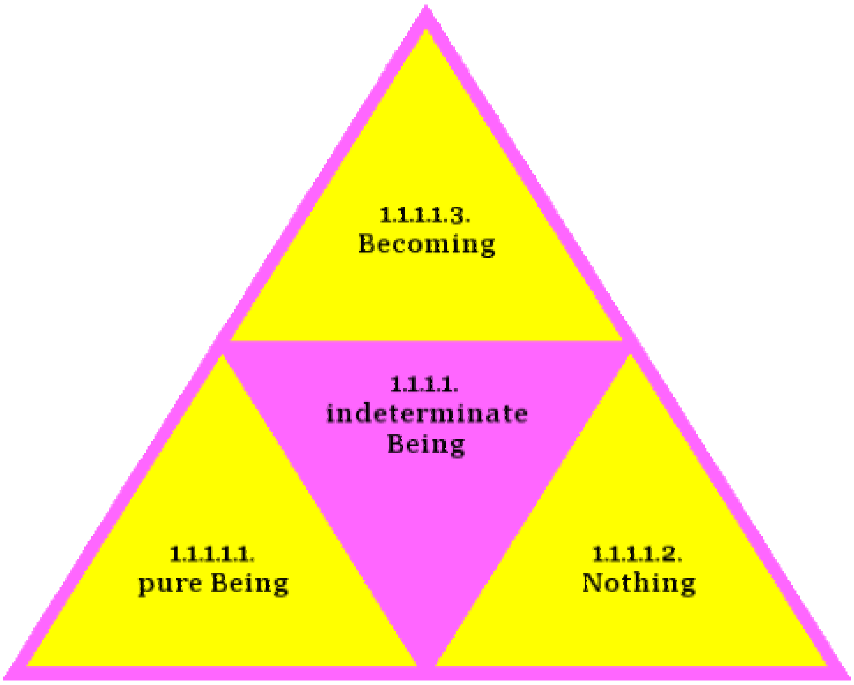
Kant’s ideas have been vastly extended by his followers Fichte and Hegel. This is truly fascinating, but is on a much less safe ground than Kant’s sober analysis. It is the philosophical counterpart of the great Romantic Movement in literature and art, which is one of the greatest moments of the human spirit.

In fact, Kant’s reasoning was completely turned around and used in a way very contrary to Kant’s intentions. Fichte made Kant’s “abstract transcendental subject” into an all-compassing “absolute subject” from which there was only a step to Hegel’s comprehensive elaboration. (Cf. CO2 and the “Fichte iteration” in the book SMU, p.220).The criticism Fichte received was severe: he was accused of “atheism” and removed from his university job. In fact, his method can at best lead to a God immanent in the world, a refinement of Spinoza’s pantheism into a panentheism, by equipping Spinoza’s infinite substance with a subject-object structure. Panentheism means “God (as the highest subject) in the world”, somewhat like Leibniz’ supreme monad in a world of monads. Panentheism, however, also implies the opposite: just as God is in the world, the world is in God. Later, Fichte tried to show the basic identity of his later system, Wissenschaftslehre of 1804, with the Gospel of St. John, and, although I do not share his optimism completely, I am impressed by his enthusiasm, the beauty if his ideas, and his obvious sincerity. Still, he does not seem to be able to arrive at a transcendent God

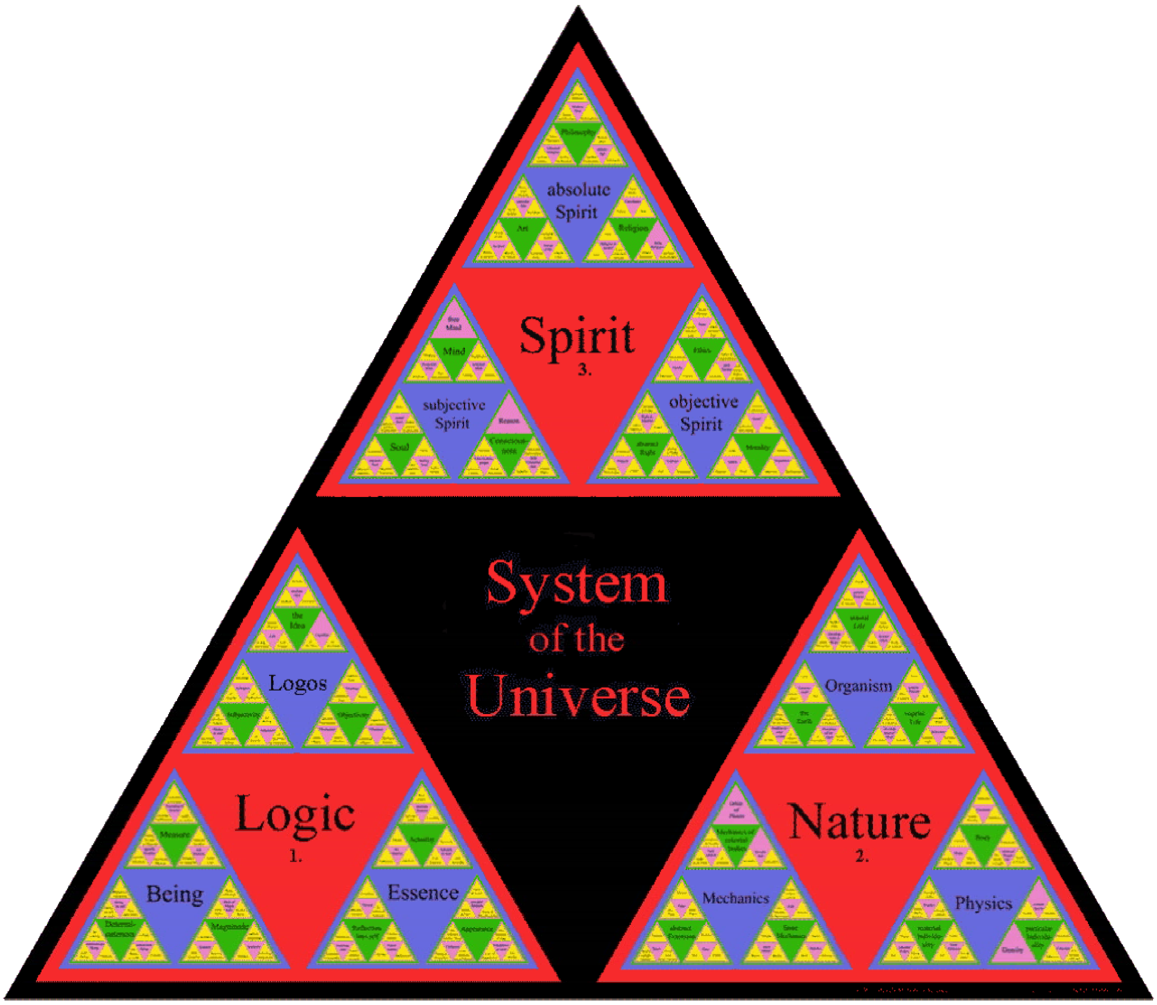
Hegel

Hegel boldly generalizes Kant’s subject-object structure of knowledge from man to God as the Absolute Subject. With Aristotle, Hegel says hat the Absolute is self-thinking thought. From there it seems to be a logical step to the Jewish-Christian-Muslim concept of a personal God. It gives an interesting interpretation to the biblical saying that God created man in His image. With Aristotle, Hegel says hat God is self-thinking thought.

Hegel starts from “pure Being”: his first triad is “being—nothing—becoming”, as the following picture shows:

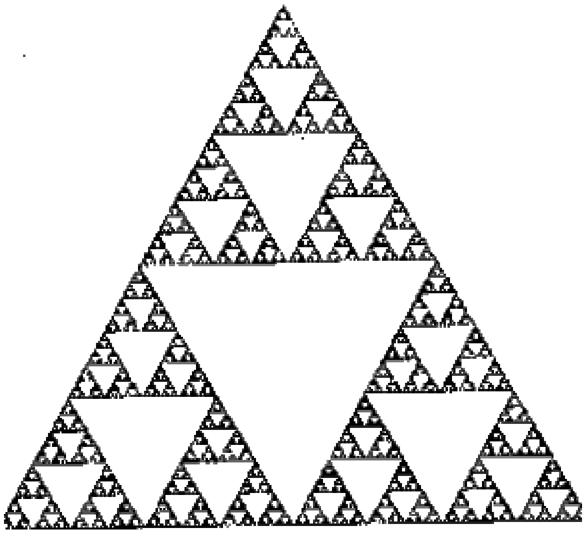


extended by Hegel to his entire system of consecutive triads. This has been ingeniously represented by Kai Froeb, Martin Grimsmann and Lutz Hansen as the



Hegel fractal, which is based on the well-known infinite *Sierpinski fractal.*

Both figures have been gratefully borrowed with permission from www.hegel.net.
The Sierpinski fractal has been



taken from www.cs.cornell.edu/Courses/cs312/2000fa/handouts/fractals.html .

But philosophy alone can merely lead to an *immanent God*, a God within the world (CO2, p. 190). Hegel's triadic progression is a logical rather than temporal process: "But the Absolute in itself does not, to put the matter crudely, start as pure Being at seven in the morning and end as self-thinking Thought at seven in the evening", CO2, p.191. (The first triangle above would correspond to "seven in the morning", and the whole Hegel fractal would be finished at "seven in the evening".)

Hegel uses (or misuses) Christian terminology as much as he can. For him, Christianity is even the "Absolute Religion". By a magnificent *tour de force* he sweepingly incorporates religion into his philosophical system, which is fascinating but must be taken with considerable caution. *Religion is certainly more than a mere item on Hegel's agenda*, represented by the two triangles in Appendix A1.3.

Nevertheless, Hegel's ideas have become very basic for modern Christian thinking, for Karl Barth, Karl Rahner, Hans Urs von Balthasar, etc., even when they do not agree with him. The problem with Hegel is that he starts "from below", that is from the world, from human thinking, and can, at best, obtain an *immanent* God. A *transcendent* God can be obtained first only "from above" by His revelation through the Bible. After that, Hegelian methods might well become applicable.

Section 15. The transcendental theology of Karl Rahner

Like Hegel, also Rahner (RA1, pp. 28–33) quietly bases his introduction to Christian theology on the Kantian subject-object structure of experience, following, however, a completely different approach. Karl Rahner is considered the greatest modern theologian and is highly regarded by the Church. His book, fascinating as it is, is extremely difficult to read for a modern scientist because it is very abstract and philosophical. On the other hand, Rahner's highly abstract thinking is of great appeal to a modern physicist used to the extreme abstractness of modern theoretical physics (relativity and quantum theory, elementary particle physics).

His difficult "Grundkurs des Glaubens " (RA1) has been translated into many languagues, also into English. For the German-speaking reader, the "Karl Rahner Lesebuch" (RA2) provides much easier texts; for the English-speaking reader, "The Cambridge Companion to Karl Rahner" (RA4) is recommended. There is a huge, informative and well-readable literature in the Internet. We also recommend RA4 and SCH.

Rahner took seriously the biblical quotation that God created Man according to his image. Thus He created him as a person with a subject-object structure, each individual separately. (This does not mean that man is essentially divine and "like God"!) *This is compatible with a transcendent God*. Therefore it is also called "transcendental revelation".

Thus, God left a trace of his creation; figuratively speaking, he left his visiting card. *This trace is automatically part of each mind's a-priori structure.* This is particularly pronounced in mystics, but according to Rahner, should be recognizable, however faintly, by every person who does not intentionally close himself off against God.

Rahner and Hegel.

Broadly we may say:

Hegel generalized Kant's "transcendental subject" to an „Absolute Idea". So to speak, this is an "upward movement". As we have seen in Section 14, it starts from "pure Being" and goes through consecutive and more and more complicated triads to end as "self-thinking Thought".

Rahner starts with an image of a *transcendent God* in the human soul. This is a "downward movement" from God to man "whom He created in His image", made possible by revelation. See also RA5, p.179.

Philosophy and theology: the two-floors analogy

In the book of Michael Schulz (SCH, p.89 ff.), I found the analogy of the problem with a two-floor building. Philosophy (immanence) occupies the ground floor, and religion (transcendence) occupies the upper floor. The problem is to find a connection between the two floors, which is the business of theology. Hegel occupies a splendid apartment in the ground floor, without apparently being able to find a way to the upper floor. Roman-Catholic "school theology" considers Divine revelation (Bible, dogmas etc.) to be the only connection, but unfortunately it is only a one-way street downstairs. With the help of Kant, Rahner tried to construct a secret corridor (the terminology is mine) from the ground-floor upstairs.

In his triadic system, Hegel thought he had constructed also a three-dimensional path covering both floors, but in fact, his path seems to be only a two-dimensional projection onto the ground floor.

Another analogy perhaps also helps illustrate Rahner's idea.

Section 16. A geometrical analogy

According to classical theology, God *transcends* nature including human thinking, but is also *immanent* in it through the beauty of His creation.

For a first simple, even primitive, introduction we may take the elementary circle model of Section 5, using the curious formal correspondence

God ←→ mathematics
man ←→ physics

Although first it looks rather artificial, this "analogy" works quite well. The statement "mathematics cannot be proved, but made plausible, by physics" would correspond to the statement "God's existence cannot be proved, but made plausible, by philosophy".

In the geometrical analogy, God would correspond to the ideal mathematical circle, and the human mind would correspond to the "fuzzy" physical circle.

By this analogy, God's immanence in human mind (at least his "visiting card") as presupposed by Rahner (see the preceding section) would appear rather plausible. Rahner teaches his students theology in much the same way as a good teacher explains geometry by drawing figures on the blackboard.

Rahner (RA1, p.64 ff.) thinks that everybody should be able to find God in the depth of their own soul. Guided by our primitive analogy, we may venture to say that the soul reflects God, in somewhat the same way as the "fuzzy" circle reflects the "ideal" circle. This is rather daring, isn't it, but the heuristic picture might help.

Of course, the circle is a poor and stupid picture, and it is really totally inadequate. As an excuse: the circle has been regarded as the most perfect curve already by the Greek thinkers… (If you think that this reasoning sounds Platonic, you are perfectly right; see Appendix 2.)

Section 17. Gödelian thinking again

All this is not new. The immanence of God in our thinking has been standard for the great mystical thinkers of all religions. An "*analogia entis*" (analogy of being) has been used by medieval philosophy and classical Thomist theology. It gives an interesting interpretation to the biblical saying that God created man in His image.

Finally, Hegel's thinking loses much of its provocative appearence if it is considered an interesting and very instructive chain of dialectic reasoning rather than an absolute truth.

Like almost all philosophical and even scientific thinking, theological reasoning is "non-algorithmic" and principally "non-rigorous". In theology there in an additional problem: the "object" (God) is principally infinite, so that ordinary logic is not really applicable. Strictly speaking, one cannot speak about God at all! Formulations, even in religious dogmas, must use language which is finite and apparently both over-abstract and over-concrete, for example problems with "defining" the internal trinity structure of God, as pointed out admirably by Rahner (RA1, p. 139). A certain amount of dialectic thinking (such as Hegel's thinking) is unavoidable and has permanently affected philosophical terminology.

The dogmas cannot be used as self-consistent axioms from which all theology is deduced. (By Gödel, this is already true for mathematics itself.) The dogmas have been developed over a time-span of 2000 years from different philosophic backgrounds. So it might be awkward if the wording of the dogmas is taken literally: they should be carefully interpreted to smooth their ragged edges, in which Karl Rahner is an unsurpassed master. Religious revelations, coming from God, may be errorless, but as soon as they are cast into words, they necessarily become affected by the imperfections of our poor human languages. Pointedly this can be expressed as follows: "A truth, if expressed in words, becomes a lie." (I have read this fine "Platonic" statement ("Мысль изреченная есть ложь") in the poem "Silentium" (1830) of the Russian poet F. I. Tyuchin.) See also Appendix 2.

Uncertainty has been seen to occur in the observation of natural science (Gauss), in physics (Heisenberg) and even in mathematics and logic (Gödel). So a basic uncertainty in "informal" philosophical and theological thinking would appear only natural.

Good informal reasoning is "uncertain, heuristic, plausible, approximate", but not "wrong". Furthermore, even in mathematical and physical work, creative thinking is usually informal, and axiomatization comes much later if at all.

Section 18. Tolerance

If I strongly believe in something, why should I take seriously the different belief of others? This is the old problem of tolerance.

There are basically five main world religions: Judaism, Christianity, Islam, Hinduism and Buddhism.

Judaism is fundamental for Christianity, and both are essential for Islam. The Jewish Torah and other Jewish books constitute the Old Testament, the first part of what Christians call the Bible. The second part of the Bible, the New Testament, contains the teachings of Jesus and is specifically Christian. The Islam also has a Holy Book, the Koran. *All these books are considered Divine Revelations by their followers*. All the religions are monotheistic and worship the same God.

The Eastern religions are Hinduism, primarily in India, and Buddhism, mainly in the countries east of India. Buddhism has developed out of Hinduism, somewhat like Christianity has developed out of Judaism. (Hinduism really comprises several related Indian religions.) The Eastern religions are much more complex and difficult to characterize than the "Western" religions (Judaism, Christianity and Islam). We may perhaps say that the "Western" religions are primarily monotheistic with elements of mysticism, and that the Eastern religions are primarily mystic with monotheistic, as well as polytheistic (in Hinduism) and even "atheistic" (in Buddhism) elements.

Let us begin with some general facts:

- Each religion holds mainly (but by no means exclusively) in certain geographical regions.
- Each religion corresponds mainly (but not exclusively) to a certain culture.

- All world religions have passed the test of time: all are more than thousand years old, and they will certainly continue to exist in the future.
- Each religion embodies enormous cultural, moral and spiritual values.
- Each religion considers itself unique and usually superior to the others.
- Conversions between them are rare and have been achieved mostly by force.

So these world religions will stay with us, in spite of migration, social changes and globalization. They will come closer to each other geographically, politically, and as a consequence of civilization. For a peaceful coexistence between these religions, inter-religious dialogues in a climate of mutual understanding and respect are urgently necessary. Such dialogues are a chance to learn from each other, to broaden one’s horizon and, eventually, even to understand one’s own religion better.

Our thinking is affected by many uncertainties as we have seen. A positive outcome of this basic uncertainty of thinking should be the acceptance of *pluralism in philosophy* and of *tolerance towards different religious beliefs.*

It is tempting to consider various religions as projections of the perfect Divine Truth on different ways of human thinking, which is necessarily limited by the imperfections of language. (These different ways of human thinking might correspond to different cultures.) This is an interesting idea which, however, seems to carry relativism too far. All religions have similar elements, but they are by no means “basically identical”. The question of truth cannot be disregarded.

I am not qualified to treat this matter in any detail. The relation between religions and cultures and other relevant problems are treated profoundly in RZ1 from a Christian point of view.

A clash of cultures and religions can be regarded either as a disaster or a challenge. The history of Europe has been shaped by such clashes.

The possibility and necessity of peaceful and respectful coexistence and cooperation is shown by the example of South-East Europe, in spite of the recent war. Especially in Bosnia and Herzegovina, people with different religious beliefs must live together in mutual tolerance.

A possibility to do this with intellectual honesty might be a combination of Gödelian uncertainty and biblical wisdom (“Do not judge others”, Matthew 7:1–5): My poor thinking may be sharp enough to serve as a base for my own belief, but it is certainly not sharp enough to serve as a base for my judging the beliefs of others.

Let me finish with my favorite quotation from Hans Urs von Balthasar: “*Die Wahrheit ist symphonisch*”, Truth is symphonic.

APPENDIX 1. Simplified logical structures

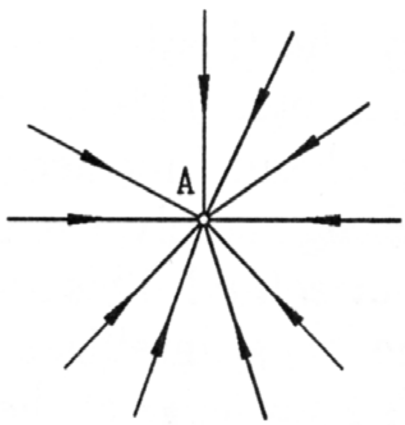
In some kind of mathematical but very loose terminology, a monadic, a dipolar and a triadic structures are related to each other like a point, a dipole, and a triangle, somewhat like

```
monad     o


          o
dipole    I
          o


           o
triad     / \
         o – o
```

These pictures are very primitive but should give an intuitive idea of what we are going to discuss.

A1.1. Monadic structures

The concept of *monad* was introduced by the great mathematician and philosopher Gottfried Wilhelm von Leibniz (1646-1716). A monad is essentially a “point-like” subject *A* mirroring the whole universe, that is, *A* is a center of perspective for all objects. Thus a monad is considered a point-like subject *together with all objects as seen from A.* Symbolically we may represent a monad perhaps somewhat as in the following figure



According to Leibniz, there is a monad at each point of space. Human persons (souls) are particularly well-developed monads, and God is the greatest monad of all. All “monads” are windowless, that is, closed to each other and without interaction with each other except with God.

Another mathematician and philosopher, Alfred North Whitehead has worked out a theory of monads which do, in fact, interact. (Whitehead is quoted several times in this paper.)

A1.2. Dipolar structures

The name is taken from magnetism: a *bar magnet* has two poles, a North pole and a South pole. The usual subject-object structure is a dipole structure *A*—*B* or rather

$A \leftarrow B$

A denotes the subject and *B* is the object. The arrow denotes that object *B* is observed by the subject *A*.

To indicate the fact that, as noted by Kant, also the observer *A* influences the object *B,* we may use the diagram

$A \leftrightarrow B$

If we consider Fichte, we also have the dichotomy

I — Non-I,

where "I" denotes the subject interacting with the World modestly called "Non-I".

The same diagram also may be use for the relation God—Word:

God — World

(the analogy with Fichte is not accidental). To indicate a *transcendent* God, Creator of the World, we may use the symbol

God → World,

and a *transcendent and immanent* God (responsive to our prayers) corresponds to

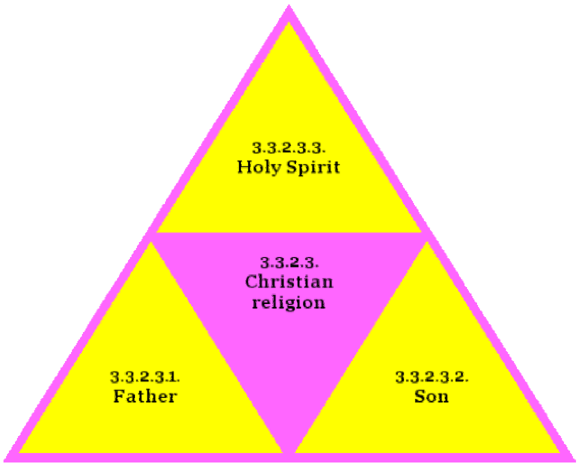
God ↔ World.

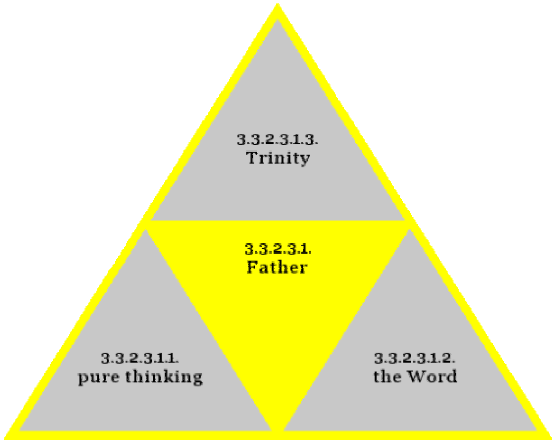
(This primitive symbolism would indicate that God is a finite object which emphatically is not the case.)

A1.3. Triadic structures

The Original Triadic Structure is, of course, the Divine Trinity (Father, Son and Holy Spirit). Even theologians find it very difficult to express the unity of ONE GOD in three Persons. Here I must be silent. A very interesting modern discussion is found in SCH p.160 ff. On p. 174 we read "In a similar way, Hegel understood the Trinity as Absolute Subjectivity going through three steps (as Father He is the absolute subject simply as such, as Son He is the absolute subject differentiating Himself as Logos, and as Spirit He is the absolute subject as unifying relation between Himself as Father and Son)”. (The poor translation is mine, the obscurity is typically Hegelian.)

Hegel’s ideas are popularly expressed in two triangles from the “Hegel fractal” taken from www.hegel.net with permission:





Plotinus.

Plotinus (204-270), following Plato (Timaios 31b -32) and Aristotle’s definition of God as self-thinking thought), has given a very interesting triadic structure (Ennead V 3):

The thinking thinks the thinking.

which is obviously true. Here the logical subject (thinking) is identical to the object (thinking) and to the predicate (thinks). It is a highly non-algorithmic: no imaginable computer language would accept it as a formula. It is perhaps the most striking dialectic structure (see Section14). These Platonic ideas have certainly influenced the theory of the Christian Trinity. (In his “Confessions”, St. Augustine has acknowledged his debt to Plotinus.)

Triads in philosophy.

Since then the study of the Trinity has been one of the most fertile sources of philosophic thinking, up to German Idealism: Fichte and, particularly, Hegel, as we have seen in Section 14 (remember the colourful pictures!). Hegel consistently thinks in triads (the Marxists have followed him in this). Even the sober Kant has tried to arrange everything in triads: especially the three Critiques, which, so to speak, correspond to the famous Platonic triad of “truth, goodness, beauty”. Even A. N. Whitehead has remarked that the theory of the Trinity has been an ancient example of what he calls “internal relations”.

A legend about St. Augustine

St. Augustine once walked along the sea-shore, meditating about the Divine Trinity. He saw a little child drawing water from the sea by means of one of the shells lying around. Augustine asked: “What are you doing?” The child answered: “I am emptying the sea”. Augustine said: ”But this is impossible!” The answer of the child was: “It is as impossible as your trying to encompass the mystery of the Divine Trinity by your thinking.”

APPENDIX 2. A contemporary Platonic world view

Some kind of Platonism seems to be a typical philosophy for many mathematically-minded persons. Gödel and Penrose, even Russell, might be considered in this context. It was Alfred North Whitehead who said that “the safest characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato”.

Nonverbal thinking, language and logic

Contrary to a common opinion, thinking is largely *nonverbal*, that is, not using words. I happen to speak several languages. If I am asked in which language I am thinking, I answer: “In no language at all, except, for instance, when I am preparing a lecture in a certain language. In this case I think about formulations in the language of my lecture.” Otherwise I think nonverbally, in vague and indistinct images, structures and associations.

This may sound strange, especially to a non-mathematician. In his book “*The psychology of invention in the mathematical field*” (Princeton Univ. Press 1945), the well-known French mathematician J. Hadamard has investigated this on the basis of experiences of people such as Albert Einstein and Henri Poincarė. When such people think creatively, they think intuitively and in vague images and structures. Only after they have discovered a mathematical theorem, they must supplement the logical proof, in order to convince themselves and communicate their results. (I can only confirm this by my own experience.) More about nonverbal thinking may be found in SMU, pp. 30-32.

This should by no means diminish the importance of language. It is the basis for this vague intuitive nonverbal thinking. The structure is probably very much the same in most languages; this is what makes translation possible, at least approximately. Translation works well in scientific texts, whereas poetry and even philosophy is much less easy to translate, as the Italian proverb, “*Traduttore – traditore”,* expresses.

Thus it seems that linguistic formulations play, in philosophy or history, a much larger role than in mathematical sciences. This may be one of the reasons why English, as a *lingua franca*, is commonly used in natural sciences, and less so in the humanities.

Using a mathematic analogy, our informal thinking is a *continuum* of ideas, feelings (Whitehead’s favourite expression), and shades of meaning. Translating it into ordinary sentences of *discrete* words can be called a *discretization*, not unlike a measurement which always transforms a “true value” which is usually an irrational number (in a mathematical sense) into an observed value consisting of a finite number of digits only. Another similar expression is *digitalization* familiar from computers.

Furthermore, the nonverbal continuum of intuitive thinking freely mixes *logical levels,* see Section 9, which makes it so suitable for dialectic thinking. (For instance, when I write a sentence, I may, at the same time, think whether this expression is suitable or should be corrected, which is “thinking about thinking” on a higher level.)

To repeat: putting thinking into words is some kind of projection on a certain linguistic structure. *This projection loses information* and introduces inaccuracies, which is comparable to measurements projected on the rational numbers and affected by measuring and round-off errors (Gauss' error theory, Section 4).

By this process, some kind of oversimplification is almost inevitable. Let me repeat a quotation in Section 17: "A truth, when expressed in words, becomes a lie."

Let us illustrate this by an example. Consider the statement: "This leaf is green". It is inevitably wrong, or at least ambiguous or imprecise. There are various shades of green, and the leaf may be overall green, but with small yellow spots.

This is a relatively simple case. In more complex situations, the distortion by language increases enormously. An extreme case is a theory of the Divine Trinity, which strongly involves infinity and corresponding antinomies.

An example of physics is the duality of elementary particles (electrons as particles of electricity, photons as particles of light). Electrons may usually be regarded as particles, but sometimes behave as waves. Light usually behaves as a wave, but sometimes it behaves as particles. This is Einstein's famous discovery of photons (1905). This particle - wave duality is rigorously resolved by the mathematics of quantum theory.

Thus, in comparison to the real world, mathematics seems an ideal language, infinitely precise, a Platonic world. It has good theories of infinity such as Cantor's transfinite numbers or Leibniz' infinitely small numbers, which have always been used in differential and integral calculus and recently have even be axiomatized. No wonder that, for more than 2500 years, mathematics has been recognized as a perfect "Platonic world" of exact ideas.

As we have seen, medieval Platonism (the most important was Buonaventura) has put these ideas even into the Mind of God (the Logos is the second Person of the Trinity!)

The three-world terminology of Popper and Eccles

This terminology has been introduces by the famous but controversial book "*The Self and Its Brain"* by K. R. Popper and J. C. Eccles (Springer 1977):

World 1 is the real world in which we live;

World 2 comprises our subjective sense impressions and feelings: visual impressions but also the headache from which I am suffering right now;

World 3 consists of mathematics and other Platonic ideas.

This three-world model is an extremely useful reference even for those who disagree with it. Roughly speaking, World 1 corresponds to philosophical realism, World 2 is the world of sense data so prominent in philosophical idealism but also in the neo-positivist theory of sense data, and World 3 is for the mathematicians and theoretical physicists. More about it will be found in SMU, pp. 207-213.

Are physical theories absolutely correct?

Consider *classical mechanics*, which has been naively regarded as a prototype of a precise physical theory. A hundred years ago (in 1905), however, Albert Einstein showed that this assumption was not correct since for high velocities (close to the velocity of light), classical mechanics ceases to be correct and must be replaced by the *theory of relativity*. Two decades later, Schrödinger and Heisenberg showed that in atomic and molecular dimensions, classical mechanics must be replaced by *quantum theory.*

However, relativity and quantum theory, both extremely precise, can be shown to be mutually inconsistent! Hence, *they cannot both be absolutely precise.*

For almost a hundred years, scientists, among them also Einstein and Heisenberg, have worked on a *unified theory.* Not only that nobody has found such a theory so far, the goal appears farther and farther away as time passes.

Gaussian errors (Section 4), chaos theory (based on classical mechanics!), Heisenberg uncertainties and quantum fluctuations further complicate the picture and let the determinism of classical mechanics appear like a nostalgic dream, given up long ago. Many more details can be found in SMU, pp. 80 and 100.

World 1 remains imperfect and "fuzzy", and a continual approximation by more and more precise scientific theories appears more and more improbable. We also have the principal philosophic difficulties of objective realism in the sense of Kant (Section 10).

The Platonic world of mathematics (World 3) remains relatively untouched by these problems. It has consistency problems of its own, expressed by Gödel's and related theorems (Section 8), but I would not worry that God does not know a solution to Gödel's problem.

Plato's perfect World 3

Mathematics is a subset of World 3, which has been considered "real" by the (mathematical) "realists" or Platonists from Pythagoras and Plato to Kurt Gödel and Roger Penrose.

Kant has considered mathematics as the main example of a synthetic a priori science (Section 11). In fact, not all true mathematical theorems are "provable" as Gödel showed. Not all true theorems are accessible to "algorithmic reasoning" in the sense of Penrose.

Famous examples of theorems probably found by "non-rigorous" intuition are Fermat's Last Theorem (before 1665) and Riemann's theory of the distribution of prime numbers (1859).

Fermat's theorem (see SMU p. 208 or Internet) is a logically very simple and beautiful statement about prime numbers, which are he simplest possible mathematical numbers. Its proof, however, has been tantalizingly difficult. The (hopefully) final version of the proof, around 1995, comprises hundreds of pages and is as roundabout and inelegant as possible, a brutal *tour de force.* It looks as if the theorem is simply "out there", just waiting for the discovery by Fermat and possibly still for a more elegant proof.

Riemann's theory of the distribution of prime numbers (see SMU p. 209 or the Internet) is not that simple, but is considered one of the most beautiful topics of mathematics. Riemann was one of the greatest intuitive geniuses of mathematical discovery. His paper was written in a hurry and not all of his proofs are rigorous by modern standards. It took two or three generations of mathematicians to make the proofs rigorous, and one of his important conjectures has yet to be proved (or disproved).

What we wanted to show here is that mathematics is not invented by man. It already pre-exists human mind and is discovered rather than invented. So what we have said at the end of Section 2 may not be inappropriate.

Furthermore, mathematics is incredibly perfect and extraordinarily beautiful. As we have said in Section 14, the beauty of the mathematical theories of physics *comes from the perfection of mathematics.*

Thus, Platonist philosophers believe that World 3 provides, so to speak, a perfect model of our World 1.

Our imperfect World 1

As we have seen, our World 1 contains imprecision as a necessary element. This is an imperfection, but is it bad?

According to Leibniz, God created the best of all possible worlds. This is a rather empty statement because "best" is not defined. The biblical statement is much more substantial: "God saw all he had made, and it was very good" (Genesis 1:31).

Imperfections may be beneficial. Strictly speaking, classical mechanics in its most rigorous and perfect form (such as celestial mechanics), does not admit *mechanical friction*. Absence of friction would mean that everything would be completely slippery, worse than ice: we would not be able to walk, drive a car… It would also be horribly cold, since friction generates and transports heat.

To change subject, no traffic is possible without laws, but also flexibility and tolerance are necessary for safe driving.

Our ordinary World 1 in its basic "fuzziness" seems to need some flexibility, imperfection, friction, but also tolerance, compromise etc., in order to work. Even "perfect morality" and "absolute honesty" seem unrealistic in our imperfect world: *summum ius = summa iniuria.* Christianity means justice tempered by love. Absoluteness is for dictatorships, totalitarian ideologies and religious extremisms.

Even Beethoven's and Bruckner's magnificent symphonies require the tempered musical scale introduced by Johann Sebastin Bach in order to obtain an otherwise impossible richness and expressivity in music, a scale which is based on a mathematical compromise, admitting tiny, practically inaudible impurities.

A physical expression of imperfection, disorder, friction, decay, cooling of the Sun and consequent "heat death" of our planetary system if left alone, is *entropy*, a measure of "disorder". According to physics, *the entropy of the universe increases*. (My desk full of papers, if left to itself, becomes more and more disorderly.) Biology works in the opposite direction: biological order through genetic information counteracts the disordering tendency of entropy. A biological individual grows, develops and flourishes; but finally it dies and decays: entropy taking over again.

It is rather fashionable nowadays to interpret some statements in St. Paul's Letter to the Romans as a link between entropy and "original sin" (!). This is an intriguing idea but seems to me rather far-fetched.

The "original sin" is Adam's using his free will against the will of God by eating "from the tree of knowledge of good and evil" in the paradise (Genesis 2:17). It has been misinterpreted as the beginning of the intellectual maturity of mankind, the beginning of free and independent thinking and by this, even of science. In my opinion, God did not forbid free will and free thinking, but has warned against their misuse. This is now better understood than, say, a hundred years ago. In fact, unrestricted freedom in politics has led to dictatorships, and even science and technology, positive and necessary as they are for the progress of mankind, seem to have lead to a point in which the future of mankind may be in danger.

The problem of Evil

Evil exists. It is something much worse than mere imperfection. There is the *mysterium iniquitatis* (the secret of Evil), and we cannot understand it. Christians believe that God takes the Evil very seriously and will finally overcome it, as the death and resurrection of Christ show.

Even though God did not create Evil, He seems to tolerate it for purposes which He alone knows. My attempt of explaining the Tsunami problem in the Introduction, if not wrong, is certainly oversimplified and rather amateurish. Whitehead's statement, "God is the fellow sufferer who understands", is better.

Sometimes, Evil may produce Good. A standard example is the fact that illness has been the engine of incredible progress in medicine and biology.

An exceptionally profound treatment has been given in Cardinal Lehmann's paper "Das Böse – oder das Drama der Freiheit", Materialdienst 9/03, reproduced in part in the Internet:
www.ekd.de/ezw/35583.html
Another favourite of mine Is Rahner's article on Evil in RA2, pp. 115-118.

The Evil is mentioned in the Lord's Prayer (Paternoster), but not directly in the Credo, the Christian Confession of Faith. The Evil is real, but the Good is predominant.

APPENDIX 3. Why I am a Christian

Introduction

Bertrand Russell wrote an article „Why I am not a Christian“ (1927). As a boy, I liked Russell very much as a philosopher because he writes attractively and clearly. Later I greatly appreciated his pioneering work in mathematical logic ("Principia Mathematica"). His article is still quite popular.

In fact, I recently came across this article, reprinted in the Internet www.users.drew.edu/~jlenz/whynot.html
I was interested to read it again after perhaps 50 years. Frankly, I was disappointed because it now appears rather superficial, with commonplace arguments. At least, however, it furnished the title for my present Appendix 3.

Can we prove the existence of God?

From ancient times up to Descartes and Leibniz it was considered a main business of philosophy to prove the existence of God. It is a commonplace that Kant destroyed the metaphysical illusions about such "proofs". What is a proof and what does existence mean?

In the precise language of modern logic, a scientific proof is a logical deduction from some well-defined set of axioms, such as a mathematical proof.
As we have seen in Section 13, such a formal proof cannot exist. The reason for the fact that the existence of God cannot be proved "exactly" is simply the inadequacy of our logic and of our language.

Still, the "existence of God" in some way has an intuitive meaning and is meaningful to many people, including myself.

I believe that a high degree of intelligible order is a necessary condition for a scientist's work. "After some idealization", science and even philosophy work. After all, it is not unreasonable to believe in God.

Science, philosophy and theology once more

Some people are naturally interested in music and go to concerts or even learn to play a musical instrument, which leaves others completely cold. Some persons look for a meaning of their life and thus have a natural interest in religion even if they do not care to "play a religious instrument", practicing a certain faith. Believers usually take the religion from their cultural background but nevertheless have to "practice" hard. Goethe said: "Was du ererbt von deinen Vätern hast, erwirb es, um es zu besitzen" („What you have inherited from your fathers, acquire it in order to possess it").

The thinking of natural scientists, particularly physicists, is instinctively directed towards the *general*; therefore they understandably tend to some general theism, to pantheism like Albert Einstein or to panentheism like Alfred North Whitehead. But some people feel that something more concrete is needed to fill a gap in their life, something like the Sermon on the Mount.

A historical personal God, in line with Biblical history and the person of Christ, is also more in consonance with the historical character of biological evolution. I cannot treat evolution here; I have tried to do so in SMU (p.173ff.). From the theological point of view, my favourite is RZ2.

My wife was a botanist. When were walking and she saw a particularly beautiful flower, she used to say with a smile: “Alles Zufall?” (All this is pure chance?).

Christianity has been thoroughly argued for and against by philosophy. On the basis of Plato and Aristotle, it has given rise to most of Western philosophy, which has enormously broadened our understanding, and I am grateful for this fact. It offers great intellectual treasures, for instance the theory of the Divine Trinity, which goes back to Plato’s dialectic. Or consider a simple logic structure such as Plotinus’ great formula: “The thinking thinks the thinking”. It is immediately evident but “non-algorithmic”: it cannot be programmed on a computer. Human reason can be wonderful indeed. It can even prove its own limitations, as Gauss and Gödel showed.

Great theologians, from St. Augustine (and much before) to Karl Rahner (and later) have elaborated a great and glorious edifice. Why not benefit from their work? Am I intellectually superior?

So I have tried to become a Christian.

Additional Reading

CO1 F. Copleston, *A History of Philosophy*, Search Press, London, Vol. VI (Wolff to Kant), 1960 (many editions)

CO2 F. Copleston, *A History of Philosophy*, ibid., Vol. VII (Fichte to Nietzsche), 1963 (many editions)

RA1 K. Rahner, *Grundkurs des Glaubens: Einführung in den Begriff des Christentums*, Herder, Freiburg, 1967 (many editions, English translation; *Foundations of Christian Faith*, New York, 1984)

RA2 K. Lehmann and A. Raffelt (eds.), *Karl Rahner Lesebuch*, Herder, Freiberg, 2004

RA3 H. Vorgrimler, *Karl Rahner: Gotteserfahrung in Leben und Denken*, Primus-Verlag, Wissenschaftliche Buchgemeinschaft Darmstadt, 2004

RA4 D. Marmion (ed.) *The Cambridge Companion to Karl Rahner*, Cambridge University Press, 2005

RA5 H. Schöndorf (ed.) *Die philosophischen Quellen der Theologie Karl Rahners,* Herder, Freiberg, 2005

RZ1 J. Ratzinger (now Pope Benedict XVI) *Glaube – Wahrheit – Toleranz: Das Christentum und die Weltreligionen,* Herder, Freiburg, 3rd ed., 2004

RZ2 J. Ratzinger (now Pope Benedict XVI) *Im Anfang schuf Gott—Konsequenzen des Schöpfungsglaubens,* Johannes-Verlag, Einsiedeln, 1996

SCH M. Schulz, *Karl Rahner begegnen*, Sankt Ulrich Verlag, Augsburg, 2002

SMU H. Moritz*, Science, Mind and the Universe: an Introduction to Natural Philosophy*, Wichmann, Heidelberg, 1995 (298 pp., AVAILABLE IN PDF ON THIS INTERNET PAGE FOR FREE DOWNLOAD!)

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